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A PRELIMINARY STUDY OF PREDICTION TECHNIQUES FOR AIRCRAFT CARRIER MOTIONS AT SEA

by

Paul Kaplan

Prepared for:
Office of Naval Research
Department of the Navy
Washington, D. C. 20360

Report No. 65-23

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FOREWORD

This report is the fourth in a series of reports produced under Office of Naval Research Contract No. Nonr-4186(00), concerned with "Hydrodynamic Effects Influencing Aircraft Carrier Landing Operations". Two reports have previously been issued on the subject of experimental model scale measurements of the flow disturbances in the air wake of an aircraft carrier. A third report was concerned with characterizing the carrier motions at sea in terms of frequency response and spectral properties. The present report is aimed at developing a technique for short-term prediction of ship motion time history.

ABSTRACT

Mathematical techniques for calculating ship motion time histories are developed for application to the aircraft carrier landing operation. Various methods for short-term prediction of motion time history are considered, based on both deterministic and statistical techniques. The most attractive approach for prediction purposes is the deterministic technique based on a convolution integral representation, with wave height measurements at the bow serving as the input data. A kernel-type weighting function, which operates on the input to provide the predicted motion history, is derived from ship response functions, and is shown to yield good pitch prediction up to 6 seconds ahead.

The limitations of classical statistical prediction techniques, as well as practical implementation difficulties, are exhibited. Certain aspects of recent prediction theory developments are considered for a proposed hybrid prediction technique (i.e. containing elements of both deterministic and statistical approaches) that will be compatible with the envisioned digital format of the predictor, and which will increase the prediction time. Recommendations for specific areas of further investigation are given for extending the applicability of the methods developed in this study.

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NOMENCLATURE

Symbol	
A'_{33}	Vertical sectional added mass
a	Amplitude of surface wave elevation
a_{ij}	Matrix element for vertical plane motions
B^*	Local beam of ship section
E_T	Mean square prediction error
$F(\omega_e;V,\beta)$	Frequency dependent part of spatial transport delay
g	Acceleration due to gravity
$J_T(\omega_e)$	Optimum predictor transfer function
$K_z(t),K_\theta(t)$	Kernel functions for heave and pitch, respectively
L	Length of ship
LCG	Longitudinal center of gravity
$M_w(t)$	Pitch moment due to waves, positive about the y-axis
S	Cross-sectional area of ship
T	Prediction time
$T_{z\eta},T_{\theta\eta}$	Response operator (amplitude and phase relative to wave) for heave and pitch, respectively
t	Time
u(t)	Unit step function
V	Forward speed
w_0	Vertical wave orbital velocity
x,y,z	Orthogonal axis system; x-axis horizontal, positive toward the bow; y-axis horizontal, positive to port; z-axis vertical, positive upward. Symbol (z) also used to represent heave motion relative to an equilibrium position.
x_b,x_s	Bow and stern x-coordinates, respectively

x_1	x-coordinate location of wave measurement point
$Z_w(t)$	Vertical wave force, positive upward
β	Heading angle of waves relative to ship
η_m	Wave elevation measured at a moving point
θ	Pitch angle, positive for rotation about y-axis
λ	Wavelength
ρ	Fluid mass density
$\sigma_\varepsilon, \sigma_\theta$	rms values of prediction error and pitch angle, respectively
τ_e	Effective time extent of kernel function
$\Phi(\omega_e)$	Power spectrum of random variable to be predicted
ϕ_z, ϕ_θ	Phase angles of heave and pitch, respectively, with regard to wave reference
ω	Circular frequency (rad./sec.)
ω_e	Circular frequency of encounter

Superscripts

(1)	Due to hydrostatic effects
(2)	Due to hydrodynamic effects

Other Symbols

$\vert\ \vert$	Absolute value
$\bar{\ }$	Fourier transform

A PRELIMINARY STUDY OF PREDICTION TECHNIQUES FOR AIRCRAFT CARRIER MOTIONS AT SEA

INTRODUCTION

Most of the studies of ship motion at sea, whether theoretical or experimental, are concerned with determining certain characteristics of the motions which may be described as forecasting or long range prediction of long-time averages of the random processes in question. Examples of such averages are the average "period", the significant amplitude, or a number of other similar characteristics that are associated with a particular mode of motion of a ship in a particular sea state environment. The basic technique for this manner of evaluating ship motions at sea is essentially linear superposition of the responses to the various frequency components encountered in the random seaway [1]. These methods of analysis, using either computed ship response amplitude operators or similar data obtained from model tests in wave basins, result in the motions being represented in the form of spectral densities, i.e. in the frequency domain, with no indication of the time history of the motion.

Analytical studies were previously carried out at OCEANICS, Inc. in order to provide information on aircraft carrier vertical plane motion characteristics (i.e heave and pitch) for different idealized sea states [2]. This information, in the form of frequency response curves (amplitude and phase of the different motions relative to the wave elevation at the ship CG position) and also spectral densities, was developed as a means of representing ship motion characteristics for utilization in a computer study and systems analysis of the entire aircraft carrier landing process [3]. The information developed in [2] could be used to develop an analog shaping filter which, in conjunction with a random white noise generator, would produce instantaneous motion time histories as a simulation tool. However, the time histories would only be represent-tative of the ship motions (i.e. of a particular realization of the ensemble of all possible motions with the specific spectrum determined by the shaping filter) up to the "present" time instant, which is the observation time of the motion occurrence.

In the course of the system analysis study of the aircraft carrier landing process using an optical landing system [3], a significant improvement in the entire operation (by virtue of reduced accident rate, waveoffs, etc.) was shown to exist if vertical plane ship motion time histories were predicted successfully and the information incorporated into the landing procedure. A separate analysis [4] has shown that the minimum prediction time necessary for implementation into such a system, which would account for the time necessary for performing appropriate maneuvers by the aircraft after receiving the necessary command, is 5 seconds, and prediction times greater than that amount (up to 8-10 seconds) would enhance this technique even further. It is also apparent that the use of such predictions of ship motion would be significant for incorporation into an automatic carrier landing system, since the information transmitted to the aircraft is terminated some 5 seconds prior to touchdown.

As a result of the significance of ship motion prediction as an aid in the landing process, a program was carried out in order to obtain an analytical development of the required predictor, together with a means for verifying the results by comparison with model studies in a wave tank. An outline of the techniques used for this program, and the results of the study are described in the present report. Throughout this report, which is concerned with the concept of prediction, it must be understood that this term implies a short-term prediction of the instantaneous time histories of the particular modes of motion, prior to their occurrence. The prediction is desired to be a continuous time record that can be easily compared with the actual record of the quantity itself, in order to assess the degree of prediction capability.

This work was carried out at OCEANICS, Inc. under the sponsorship of Air Programs, Office of Naval Research, Department of the Navy, under Contract number Nonr-4186(00).

DISCUSSION OF TECHNIQUES USED IN ANALYSIS

While most ship motion studies are represented in the form of spectra for particular sea state conditions, some limited studies have been devoted to the problem of presenting time histories of the motion for a particular sea condition (e.g. [5], [6]). These studies have been primarily concerned with duplication and presentation of the actual motions experienced by a ship model in an irregular wave system generated in a wave tank. The time domain representation of the motions has been based upon convolution integral operations on the wave histories obtained at some point located ahead of the bow of the ship. The weighting function operating on the waves has been shown to be the impulse response function of the ship response to a wave input, with the mathematical derivations of this form being based upon the assumption that the dynamic system represented by a ship in waves is the same as a simple mechanical (or electrical) dynamic system subjected to an arbitrary forcing function. The derivations in these cases do not adequately account for the fact that the hydrodynamic system (a ship) experiences forces due to waves that result from both a spatial distribution of the waves, as well as their time variation. This is due to the fact that when a wave impulse (with respect to time) occurs at a fixed point, there is an associated wave system present throughout all space, and the ship will experience a force due to this wave system associated with the localized wave impulse. In view of this spatial distribution of wave effects, and its attendant influence in determining forces on a ship located at some distance from a reference measuring point, an alternate derivation of the time domain representation of ship motions will be outlined in this report. The form of the resulting time domain operator, and its relation to those derived by previous investigators, will be discussed. Similarly the operations necessary for transforming a mathematical function that is used for motion reproduction (i.e. time history up to the present observation time) into one that functions as a predictor will also be presented.

The concept of carrying out predictions, when using as an input the measurements of the waves at a point located ahead of the bow of the ship, is based upon the fact that the essential forcing elements are the waves themselves, and that when these waves are measured some time in advance of their action on the ship, an appropriate prediction of the motions can be obtained. Since the waves that excite significant heave and pitch motions of an aircraft carrier are approximately in the range of .75-1.0 times the ship length, then in accordance with this interpretation a measurement of the waves at a point just ahead of the bow will provide a significant phase lead that will be useful for a prediction system. This type of heuristic argument must be verified for the particular application to the present case of an aircraft carrier, since the previous studies applied to other ships (which are smaller in length) have only been able to provide motion reproduction, with only one exception [7] where prediction was obtained for a wave measuring point located at a distance equal to half the ship length ahead of the bow. The requirements as to the appropriate location of the wave measuring point and the significance of ship size and speed on the prediction capability using this approach will be considered in this program.

In contrast to this technique of prediction based upon measurement of the waves at a point ahead of the ship, other prediction techniques will also be considered. The one most appropriate is that based on the theory originally developed by Wiener [8]. The Wiener prediction method is essentially a statistical technique, where the actual predictor is derived on the basis of knowledge of the spectral characteristics of the stochastic variable under consideration. The technique is generally known and outlined in various books concerned with random processes, e.g. [9], [10]. The technique based upon a convolution operation on the waves measured at the ship bow may be described as a deterministic method of reproducing the motion time history, and even when it is extended as a predictor the technique remains deterministic, i.e. it is independent of the statistical characteristics of the ship motion itself or the waves. An outline of the methods that must be used for each of these techniques will be given in this report, and the inherent advantages and disadvantages will be discussed. Certain "hybrid" techniques, which incorporate elements of each procedure, will also be considered, and the relative advantages of that approach will be portrayed as a conceptual method for extending prediction time. The details of these various methods will be presented in the ensuing sections of this report.

ANALYSIS OF IMPULSIVE WAVES AND RESULTING HYDRODYNAMIC FORCES ON A SHIP

In order to carry out an analysis of the properties of a wave system relative to a moving ship, a coordinate system is chosen with its origin on the undisturbed water surface at the ship LCG position. This coordinate system is very similar to that used in [2]*, with the x-axis positive toward the bow (in the direction of forward motion), the y-axis positive to port, and the z-axis positive upward. Waves on the free surface are functions of time, t, and the two space variables x and y. For the present analysis all of the waves are assumed to be traveling in the same direction (i.e. uni-directional seas), and they are observed at a point along the x-axis (i.e. $y = 0$). To account for wave motion relative to a moving ship, the frequency domain is that of the (circular) encounter frequency ω_e, which is defined by

$$\omega_e = \omega + \frac{\omega^2}{g} V \cos \beta \qquad , \qquad (1)$$

where ω is the circular wave frequency, V is the forward speed, and β is the heading angle of the waves relative to the ship x-axis. Equation (1) is assumed to represent waves in the range of head to beam seas, which are the predominant wave conditions expected to be encountered during aircraft carrier landing operations.

A wave disturbance measured at a point x_1 ahead of the translating ship bow is expressed in Fourier form as

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*The only difference is in the vertical location of the origin of axes, which will have no significant influence on either the equations or the actual heave and pitch motions themselves.

$$\eta_m(x_1,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} N_1(x_1,\omega_e)e^{i\omega_e t}d\omega_e \quad . \tag{2}$$

This same wave at a point x, which is located downstream along the x-axis (not necessarily along the direction of wave travel from x_1), is observed with a phase lag of $\frac{2\pi}{\lambda}(x_1-x)\cos\beta$, where λ is the wavelength of the wave given by

$$\lambda = \frac{2\pi g}{\omega^2} \quad . \tag{3}$$

This phase lag, or spatial transport delay, is derived on the basis of linear wave theory, and this result follows as a generalization of similar equations presented by Davis and Zarnick [11]. This phase lag can be represented as a function of ω_e, as shown by the following:

$$\begin{aligned}\frac{2\pi}{\lambda}(x_1-x)\cos\beta &= \frac{\omega^2}{g}(x_1-x)\cos\beta \\ &= \frac{(\omega_e-\omega)}{V}(x_1-x) = F(\omega_e;V,\beta)(x_1-x)\end{aligned} \tag{4}$$

since ω can be represented as a function of ω_e in accordance with Equation (1). The resulting expression for the wave disturbance at the point x is given by

$$\eta_m(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} N_1(x_1,\omega_e)e^{i\omega_e t-iF(\omega_e;V,\beta)(x_1-x)}d\omega_e \quad , \tag{5}$$

which by Fourier inversion can then be represented in the form

$$\eta_m(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)\int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e;V,\beta)(x_1-x)}d\omega_e d\tau \tag{6}$$

With the wave disturbance known as a function of time and position along the x-axis in terms of a measured value at the point x_1 (for all values of $x<x_1$), it is then possible to determine the

hydrodynamic forces acting on a ship placed within this wave field by application of a particular type of slender-body theory (known in ship hydrodynamics as strip theory). As an example, the hydrostatic vertical wave force at a local ship section is given by

$$\frac{dZ_w^{(1)}}{dx} = \rho g B^* \eta_m(x,t) \quad , \tag{7}$$

where B* is the local section beam. The total hydrostatic wave force is then found by integrating over the length of the ship that is affected by this wave disturbance, leading to a force given by

$$Z_w^{(1)} = \frac{\rho g}{2\pi} \int_{-\infty}^{\infty} \eta_m(x_1,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1} \left[\int_{x_s}^{x_b} B^* e^{iF(\omega_e)x} dx\right] d\omega_e d\tau \tag{8}$$

where x_b and x_s are the bow and stern x-coordinates, respectively, and $F(\omega_e)$ is used for simplifying the written form of the representation of the transport lag term. In a similar manner, the hydrostatic contribution to the pitch moment due to this wave system is given by

$$M_w^{(1)} = -\frac{\rho g}{2\pi} \int_{-\infty}^{\infty} \eta_m(x_1,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1} \left[\int_{x_s}^{x_b} xB^* e^{iF(\omega_e)} dx\right] d\omega_e d\tau \tag{9}$$

Considering the various additional terms that enter into the total wave force due to the hydrodynamic inertial effects, simple interpretations of these additional force components in terms of operations on the waves can be easily developed. The hydrodynamic part of the wave force, obtained by application of slender-body theory ([12], [13]) extended to the surface ship case, results in a local force on a ship section given by

$$\frac{dZ_w^{(2)}}{dx} = \rho S \frac{Dw_0}{Dt} + \frac{D}{Dt}(A'_{33}w_0) \tag{10}$$

where S is the submerged cross-sectional area, A'_{33} is the sectional vertical added mass, w_0 is the vertical wave orbital velocity, and the operator

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - V\frac{\partial}{\partial x} \qquad . \tag{11}$$

Equation (10) can be rewritten in the form

$$\frac{dZ_w^{(2)}}{dx} = (\rho S + A'_{33})\frac{Dw_0}{Dt} + VA'_{33}\frac{\partial w_0}{\partial x} \qquad , \tag{12}$$

after recognizing the resulting terms following integration over the ship length to obtain the total hydrodynamic wave force. The vertical orbital velocity w_0 can be shown to be

$$w_0(x,t) = \frac{D\eta_m(x,t)}{Dt} \qquad , \tag{13}$$

so that

$$\frac{Dw_0}{Dt} = \frac{D^2\eta_m}{Dt} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)\int_{-\infty}^{\infty}\omega^2(\omega_e)e^{i\omega_e(t-\tau)-iF(\omega_e)(x_1-x)}d\omega_e d\tau \tag{14}$$

and

$$\frac{\partial w_0}{\partial x} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)\int_{-\infty}^{\infty}\omega(\omega_e)F(\omega_e)e^{i\omega_e(t-\tau)-iF(\omega_e)(x_1-x)}d\omega_e d\tau \tag{15}$$

where the exponential attenuation to the ship CB used in [2] has been deleted for simplicity. Inserting these expressions into Equation (12) and integrating over the ship length leads to

$$Z_w^{(2)} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta_m(x_1,\tau)\int_{-\infty}^{\infty}e^{i\omega_e(t-\tau)-iF(\omega_e)x_1}\Bigg[\omega^2(\omega_e)\cdot\int_{x_s}^{x_b}(\rho S+A'_{33})e^{iF(\omega_e)x}dx + V\omega(\omega_e)F(\omega_e)\int_{x_s}^{x_b}A'_{33}e^{iF(\omega_e)x}dx\Bigg]d\omega_e d\tau \tag{16}$$

and the total wave force is given by the sum of Equations (8) and (16), viz.

$$Z_w = Z_w^{(1)} + Z_w^{(2)} \quad . \tag{17}$$

Following the previous derivations, it can be shown that

$$\frac{dM^{(2)}}{dx} = -(\rho S + A'_{33})x\frac{Dw_0}{Dt} - VA'_{33}(w_0 + x\frac{\partial w_0}{\partial x}) \quad , \tag{18}$$

so that the total hydrodynamic wave-induced pitch moment is

$$M^{(2)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \eta_m(x_1,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1}\Bigg[\omega^2(\omega_e) \cdot \int_{x_s}^{x_b} x(\rho S + A'_{33})e^{iF(\omega_e)x}dx + V\omega(\omega_e)F(\omega_e)\int_{x_s}^{x_b} xA'_{33}e^{iF(\omega_e)x}dx - iV\omega(\omega_e) \cdot \int_{x_s}^{x_b} A'_{33}e^{iF(\omega_e)x}dx\Bigg]\, d\omega_e d\tau \quad , \tag{19}$$

and the total pitch moment due to the arbitrary wave disturbance is given by

$$M_w = M_w^{(1)} + M_w^{(2)} \quad . \tag{20}$$

In all of the expressions commencing with Equation (1) up to Equation (20), it must be understood that sign $\frac{\omega^2}{g}$ = sign ω, and that sign ω_e = sign ω(for $V\cos\beta \geq 0$), so that all of the Fourier integral operations represent real functions. The x-integrations result in complex functions of ω_e, and it can be shown that the real parts are all even functions of ω_e, and the imaginary parts are odd functions of ω_e.

As a result of the previous analysis, the vertical force and pitch moment acting on a translating ship are given in terms of mathematical operations on the wave time history measured at a moving reference point (x_1) which is located at a fixed distance ahead of the ship bow. This result exhibits the precise form (based on strip theory) of the force and moment associated with an arbitrary wave disturbance that is measured prior to its "contact" with the ship, and illustrates the effects of the spatial distribution of free surface waves under the assumption of long-crested unidirectional waves producing head (or bow) seas.

SHIP MOTION RESPONSE TO ARBITRARY UNIDIRECTIONAL WAVES

The heave and pitch motions of a ship in waves are represented by the solution of the coupled equations of motion given by

$$a_{11}\ddot{z} + a_{12}\dot{z} + a_{13}z + a_{14}\ddot{\theta} + a_{15}\dot{\theta} + a_{16}\theta = Z_w \tag{21}$$

$$a_{21}\ddot{z} + a_{22}\dot{z} + a_{23}z + a_{24}\ddot{\theta} + a_{25}\dot{\theta} + a_{26}\theta = M_w \quad , \tag{22}$$

where the values of the a_{ij} are given in [2]. With the wave force and moment given by Equations (8), (9), (16), (17), (19) and (20), the solution for the motions may be represented by

$$z = \frac{1}{2\pi} \int_{-\infty}^{\infty} \eta_m(x,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1} \frac{G(\omega_e)S(\omega_e)-Q(\omega_e)H(\omega_e)}{P(\omega_e)S(\omega_e)-Q(\omega_e)R(\omega_e)} d\omega_e d\tau \tag{23}$$

$$\theta = \frac{1}{2\pi} \int_{-\infty}^{\infty} \eta_m(x_1,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1} \frac{P(\omega_e)H(\omega_e)-G(\omega_e)R(\omega_e)}{P(\omega_e)S(\omega_e)-Q(\omega_e)R(\omega_e)} d\omega_e d\tau \tag{24}$$

where

$$\left.\begin{aligned}
P(\omega_e) &= -a_{11}\omega_e^2 + ia_{12}\omega_e + a_{13}\ , \quad Q(\omega_e) = -a_{14}\omega_e^2 + ia_{15}\omega_e + a_{16} \\
R(\omega_e) &= -a_{21}\omega_e^2 + ia_{22}\omega_e + a_{23}\ , \quad S(\omega_e) = -a_{24}\omega_e^2 + ia_{25}\omega_e + a_{26} \\
G(\omega_e) &= \rho g \int_{x_s}^{x_b} B^* e^{iF(\omega_e)x} dx - \omega^2(\omega_e) \int_{x_s}^{x_b} (\rho S + A'_{33}) e^{iF(\omega_e)x} dx \\
&\quad - V\omega(\omega_e)F(\omega_e) \int_{x_s}^{x_b} A'_{33} e^{iF(\omega_e)x} dx
\end{aligned}\right\} \tag{25}$$

$$H(\omega_e) = -\rho g \int_{x_s}^{x_b} xB^* e^{iF(\omega_e)x} dx + \omega^2(\omega_e) \int_{x_s}^{x_b} x(\rho S + A'_{33}) e^{iF(\omega_e)x} dx$$

$$+ V\omega(\omega_e) \int_{x_s}^{x_b} A'_{33} e^{iF(\omega_e)x} dx - iV\omega(\omega_e) \int_{x_s}^{x_b} A'_{33} e^{iF(\omega_e)x} dx \Bigg\}$$

This result may be simplified by recognizing that the ratios of functions within the ω_e-integrals are <u>precisely</u> the complex "transfer functions" of the heave and pitch motions of a ship with respect to sinusoidal waves referred to the CG position. This may be easily demonstrated by comparison with the analytic solutions obtained from the equations of motion, wave forces, etc. developed for regular sinusoidal waves in [2]. Thus

$$\frac{G(\omega_e)S(\omega_e)-Q(\omega_e)H(\omega_e)}{P(\omega_e)S(\omega_e)-Q(\omega_e)R(\omega_e)} \cdot \Gamma_{z\eta}(\omega_e) = \left|\frac{z}{a}\right| e^{i\phi_z} \qquad (26)$$

$$\frac{P(\omega_e)H(\omega_e)-G(\omega_e)R(\omega_e)}{P(\omega_e)S(\omega_e)-Q(\omega_e)R(\omega_e)} = T_{\theta\eta}(\omega_e) = \left|\frac{\theta}{a}\right| e^{i\phi_\theta} \qquad (27)$$

in conformity with the notation of [2], and the heave and pitch motions may then be expressed as

$$z = \frac{1}{2\pi} \int_{-\infty}^{\infty} \eta_m(x_1,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1} T_{z\eta}(\omega_e) d\omega_e d\tau \qquad (28)$$

$$\theta = \frac{1}{2\pi} \int_{-\infty}^{\infty} \eta_m(x_1,\tau) \int_{-\infty}^{\infty} e^{i\omega_e(t-\tau)-iF(\omega_e)x_1} T_{\theta\eta}(\omega_e) d\omega_e d\tau \qquad (29)$$

The transport lag $e^{-iF(\omega_e)x_1}$ can be combined with the standard transfer function referred to the CG, so that this additional lag term changes the composite term into the transfer function of the ship motion relative to sinusoidal waves at the point $x = x_1$, i.e.

$$T_{z\eta}(\omega_e)e^{-iF(\omega_e)x_1} = T_{z\eta}(\omega_e;x_1) \tag{30}$$

$$T_{\theta\eta}(\omega_e)e^{-iF(\omega_e)x_1} = T_{\theta\eta}(\omega_e;x_1) \tag{31}$$

The ship motion representations may then be further simplified to the forms

$$z(t) = \int_{-\infty}^{\infty} K_z(t-\tau)\eta_m(\tau)d\tau \tag{32}$$

$$\theta(t) = \int_{-\infty}^{\infty} K_\theta(t-\tau)\eta_m(\tau)d\tau \quad , \tag{33}$$

where

$$K_z(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_{z\eta}(\omega_e;x_1)e^{i\omega_e t}d\omega_e \tag{34}$$

$$K_\theta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_{\theta\eta}(\omega_e;x_1)e^{i\omega_e t}d\omega_e \quad , \tag{35}$$

and it is understood that $\eta_m(\tau)$, $K_z(t)$, $K_\theta(t)$ are specifically associated with measurements made at and/or referred to the point $x = x_1$. The kernel functions $K_z(t)$ and $K_\theta(t)$ are defined in Equations (34) and (35) as inverse Fourier transforms of the ship motion transfer functions $T_{z\eta}$ and $T_{\theta\eta}$ (relative to the wave motion at the point $x = x_1$), and as such they may be considered as the effective impulse response functions for the ship heave and pitch motions, analogous to the same function for mechanical or electrical dynamic systems (e.g. see [9], [10]). The basic representations of ship motion given by Equations (32) - (35) are essentially the same as those derived in previously cited references ([5], [6], [7]), but they have greater applicability and are more precise in certain respects than the previous references. In particular the results are applicable to oblique unidirectional irregular waves (ranging from head to beam seas); the precise form of the spatial dependence of the wave vertical force and moment is presented; special distinction is made between the basic wave frequency and the frequency of encounter with the waves; and the use of constant coefficient differential equations for the present case of an aircraft carrier (see [2] for discussion of this mathematical model) avoids certain difficulties inherent in applying frequency-dependent coefficient equation systems to arbitrary motion analyses. Some of these factors may be relatively minor, but they allow an orderly development of the fundamental relations and provide guidance to less-experienced analysts in carrying out the required operations. Although the preceding results have been derived in a more precise manner than previous analyses, they are still limited by the defects inherent in the basic theoretical model, viz. strip theory. The final validity of the technique must be demonstrated by application and comparison with experimental data. Fortunately the previous studies have shown success in this regard, and it is anticipated that similar results for duplicating ship motion (i.e. a time history valid up to the present time instant) can be obtained for the present case of an aircraft carrier in random waves.

For application to the aircraft carrier, the only data available for evaluation of the impulse response functions was the collection of theoretical results derived and presented in [2].

Since the final measure of the effectiveness of motion time history reproduction is obtained by comparison of the results of Equations (31) - (35) (with wave motion time history at a point ahead of the bow as the input) with experimental ship motions, it is necessary to insure that the impulse response functions are valid for the test conditions. An experimental program was conducted at the Massachusetts Institute of Technology Ship Model Towing Tank [14] in order to obtain ship response characteristics in regular head sea waves, as well as ship motion time history data (and wave input data) for irregular wave conditions. The tests were carried out in head seas over a range of speeds from 10 to 30 knots, with most of the data obtained at speeds of 20 and 30 knots. The regular wave tests included wavelengths ranging from $.50 \leq \frac{\lambda}{L} \leq 2.0$, while the irregular wave tests were conducted in wave systems that were chosen to have the same significant wave heights as Sea State 5 and 6 [15] and encompassing a frequency range sufficient to excite significant ship motions. The ship model was that of the USS FORRESTAL, constructed at a scale ratio of 1/144, and the characteristics of the full scale ship are described in [2] and [16]. The waves were measured by a sonic transducer placed at a point equivalent to 50 feet (full scale) ahead of the forward perpendicular, which is within the range of practical distance in view of the large flight deck overhang. A comparison of the theoretical and experimental amplitude responses for pitch and heave, for the forward speeds of 20 and 30 knots, is shown in Figures 1 to 4, where the theoretical results were obtained from [2]. It may be seen that the theory for pitch shows good agreement with the experiments, while that for heave shows significant departures, especially for the 30 knot forward speed. No detailed phase comparisons are shown, but checking the data indicated that phase agreement was fairly good for the same modes of motion. In view of the good agreement in the case of pitch, it is expected that the impulse response function $K_\theta(t)$ obtained from the theoretical response characteristics given in [2] will provide good results in portraying the pitch motion time history, while it is not expected that such good agreement will be obtained for the case of heave motion. It can easily be shown that the amplitude responses of the heave and pitch are even functions of the frequency of encounter ω_e, while phase angles are odd functions. In view of this the expressions for the impulse functions can be written as

$$K_z(t) = \frac{1}{\pi} \int_0^\infty \left|\frac{z}{a}\right| \cos\left(\omega_e t + \phi_z - F(\omega_e)x_1\right) d\omega_e \tag{36}$$

$$K_\theta(t) = \frac{1}{\pi} \int_0^\infty \left|\frac{\theta}{a}\right| \cos\left(\omega_e t + \phi_\theta - F(\omega_e)x_1\right) d\omega_e \tag{37}$$

Numerical computations of these functions for the case of head seas were carried out by means of a digital computer using the theoretical expressions for the frequency response characteristics given in [2], and representative results are shown in Figures 5 to 7. It can be seen that the impulse (kernel) functions have values for both positive and negative values of their argument, which thereby differs from results obtained for the impulse response function derived for linear dynamic systems ([9], [10]). In the case of a dynamic system the input is a force, which produces the response motion, and the system has no response prior to the application of the input. With the response represented in the form of a convolution-type integral similar to that shown in Equations (32) and (33) (with the force input in the place of η_m), the impulse response function must be identically zero for negative values of its argument since the "effect" (system motion) cannot precede the "cause" (force input). This requirement of zero values of the impulse response function for negative time is known as the condition of physical realizability, since values for negative values of the kernel argument would lead to the conclusion that response at the present instant requires knowledge of future values of the input, which is not correct. The kernel functions in the present case of a ship with a wave record as the input are not physically realizable in this sense since the wave does not "cause" the ship motion, but the force and the moment associated with the wave are the causitive inputs. This interpretation has been presented and discussed in other studies (e.g. [6], [11]) where the hydrodynamic relations between wave and force, as well as the effect of the spatial variation of gravity waves, have also been considered in arriving at this explanation.

A particular application of the convolution integral representation of pitch motion time history, using the entire range of values of $K_\theta(t)$ (including the values for negative arguments), was carried out for a Forrestal class carrier at conditions equivalent to a 20 knot forward speed in Sea State 6 head waves. Good reproduction of the pitch motion was found for this case, but the motion was only determined to "present" time by operating on wave data about 5 seconds (full scale) in the "future" of the required motion observation time. This is due to the nature of the kernel function which is not physically realizable. Since the magnitudes of the pitch kernel are small in the domain of negative time, the kernel was modified by neglecting the part for negative time and thereby obtaining a physically realizable kernel function which is only defined for positive values of its argument. Computation of the pitch motion time history for the same conditions, using this new kernel

$$K_\theta(t) = \begin{cases} K_\theta(t) & , \quad t > 0 \\ 0 & , \quad t < 0 \end{cases} \tag{38}$$

resulted in motion reproduction which was also in good agreement with the experiments, as shown in Figure 8. The numerical values were also quite close to those obtained for the complete kernel (including values for negative time), and hence it appears that adequate motion time history reproduction can be obtained with a physically realizable kernel. An important aspect of this result is that only past and present wave input data are required to obtain motion data up to the present time instant.

The same technique was applied to the case of the heave motion for the same test run used for the pitch motion reproduction, where the physically realizable heave kernel was obtained from Figure 7 (with $K_z(t) = 0$ for $t < 0$). The results of this computation are compared with the experimentally measured values in Figure 9, and it can be seen that the motion time history reproduction does not match the experimental values as well as in the case of pitch motion. Another computation, for a different speed and sea condition, exhibited even poorer argeement, and hence it does not appear that the time history of heave is reproduced adequately by use of the available data applied to this method. Since the heave transfer functions, as shown in Figures 3 and 4, do not exhibit good agreement with experimental values, it therefore follows that the kernel functions computed from these transfer functions will not be appropriate for accurate heave motion time history reproduction. It is absolutely essential that the transfer function characteristics used in determining the kernel function be the actual transfer characteristics of the particular mode of ship motion, i.e. in agreement with measured transfer function characteristics, in order for this technique to be successful. In view of this area of disagreement, which is expected to be corrected when proper transfer function characteristics are obtained (from experiment and/or modified theory), no further consideration will be given to the treatment of heave motions in the remaining portion of this report. Since the carrier stern ramp excursions are predominantly due to pitch motions, consideration will be devoted to that particular mode of motion at this time in order to judge the future applicability of the techniques developed in this study. In view of the preliminary nature of this program, this is considered adequate for the present time.

DEVELOPMENT OF A PITCH PREDICTOR KERNEL FUNCTION

The results obtained in the previous section of this report have indicated that adequate pitch time history reproduction can be obtained by operating on the wave time history measured at a point ahead of the ship bow in random head seas. This information is obtained for the "present" instant of time by operating on the present and past history of the wave input data, by use of a physically realizable kernel obtained by neglecting small values of the kernel magnitude for negative values of its argument. Following this approach, which was initially demonstrated in [7], it appears plausible that neglecting part of the pitch kernel function for a small segment of positive time from $t = 0$ up to some value $t = T$ (i.e. by replacing the kernel function $K_\theta(t)$ by $K_\theta(t) \cdot u(t-T)$, where $u(t-T)$ is the unit step function) can provide some sort of prediction of the motion up to T seconds ahead of the present time. This neglect of the kernel function for a portion of positive time will be referred to in the remaining portion of this report as a "truncated" kernel, where the truncation is performed on the left-hand (i.e. small time) end of the curve.

The capability of prediction by this approach can be demonstrated mathematically by expressing Equation (33) in an alternate form, viz.

$$\theta(t) = \int_{-\infty}^{t} K(t-\tau)\eta_m(\tau)d\tau \qquad (39)$$

since $K(\tau)$ is physically realizable (see Equation (38)). For practical considerations it can be seen that the pitch kernel functions effectively terminate at some finite value of time, τ_e, which is of the order of 36 seconds (full scale) for a Forrestal class aircraft carrier (see Figures 5 and 6). Thus the convolution operation of Equation (39) requires a proper weighting of the past history of the input $\eta_m(\tau)$ for a time extent of τ_e seconds, which is expressed by

$$\theta(t) = \int_{t-\tau_e}^{t} K(t-\tau)\eta_m(\tau)d\tau \quad . \qquad (40)$$

This last expression yields the value $\theta(t)$ at present time t, with input data at time t and for the past τ_e seconds. If the kernel function is truncated, as described above, it is only necessary to operate on (τ_e-T) seconds of the past history of $\eta_m(\tau)$ to obtain a good approximation to $\theta(t)$. However the maximum allowable information on the past history of $\eta_m(\tau)$ is available, i.e. past history up to τ_e seconds in the past, so that operation on the additional T seconds of wave input available will yield a value of $\theta(t+T)$ which is a deterministic prediction for T seconds ahead of present time. This may be expressed mathematically after recognizing that the truncated kernel can be represented by

$$K(t)u(t-T) = \begin{cases} K(t+T) & , \quad t > 0 \\ 0 & , \quad t < 0 \end{cases} \quad , \qquad (41)$$

which leads to

$$\theta(t+T) = \int_{t-\tau_e}^{t} K(t+T-\tau)\eta_m(\tau)d\tau \quad . \qquad (42)$$

In order to check the prediction capability of the truncated kernel method described above, comparisons of measured and predicted pitch motions were made using the experimental data obtained in [14]. The results of this comparison are shown in Figures 10 to 12 which are appropriate to head seas at speeds of 20 and

30 knots, for different simulated sea conditions. Since the data is obtained from model tests, the time scale is reduced by a factor of 12, and hence the actual full scale prediction times are obtained by multiplying the scaled prediction time by this factor. It can be seen from these figures that a prediction time of just under 6 seconds, with adequate prediction capability, is obtained by application of this procedure. According to the results of [4], this prediction time is slightly more than the minimum required useful prediction time, and the analysis of the rms prediction error for these three records results in a relative error in the range from .36 to .49, where the relative error is defined to be the rms error divided by the rms amplitude of the actually measured variable, viz. $\frac{\sigma_\varepsilon}{\sigma_\theta}$. The values obtained for the pitch prediction in this preliminary study indicate an appreciable gain in the safety of aircraft landing operations according to the analysis of [4].

Since a good prediction of the pitch motion has been obtained using the truncated kernel function, it is important to determine the basis for this good performance. The significant point discussed earlier in this section is the fact that good motion time history reproduction could be obtained with a truncated kernel function ($K_\theta(t) = 0$, $0 \le t \le T$) using T seconds less of wave input data, and hence it is important to see what characteristics of the ship response are responsible for this result. The method for analysis will be by examination of the frequency response characteristics of the ship motion, which can be obtained by applying a Fourier transform to Equation (33), resulting in

$$\bar{\sigma}(\omega_e) = \bar{K}_\theta(\omega_e)\bar{\eta}_m(\omega_e) \tag{43}$$

where the bar symbols over the variables represent the Fourier transforms of the particular variable. Assuming that the pitch kernel is physically realizable, it may be represented in the form

$$K_\theta(t) = G(t)u(t) \tag{44}$$

and the Fourier transform of this kernel is then given by

$$\bar{K}_\theta(\omega_e) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \bar{G}(\nu)\,\frac{d\nu}{\omega_e - \nu} \quad , \tag{45}$$

using the theorem for the Fourier transform of a product of two time functions [17] and the fact that the Fourier transform of the unit step function u(t) is given by

$$\int_{-\infty}^{\infty} e^{-i\omega_e t}\,u(t)dt = \frac{1}{i\omega_e} \tag{46}$$

The result given by Equation (45) is to be interpreted as a principal value integral and is essentially a Hilbert transform. The truncated kernel function can be represented by

$$K_\theta(t) = G(t)u(t-T) \tag{47}$$

and its Fourier transform is given by

$$\begin{aligned}\bar{K}_\theta(\omega_e) &= \frac{1}{2\pi i}\int_{-\infty}^{\infty} \bar{G}(\nu)\,\frac{e^{-i(\omega_e-\nu)T}}{\omega_e-\nu}\,d\nu \\ &= \frac{e^{-i\omega_e T}}{2\pi i}\int_{-\infty}^{\infty} G(\nu)\,\frac{e^{i\nu T}}{\omega_e-\nu}\,d\nu\end{aligned} \tag{48}$$

where the Fourier transform of the "delayed" unit function is given by

$$\int_{-\infty}^{\infty} e^{-i\omega_e t}\, u(t-T)dt = \frac{e^{-i\omega_e T}}{i\omega_e} \tag{49}$$

Comparison of Equations (45) and (49) shows that for small values of T, there is not much difference in the transforms of the kernel function for small values of the frequency, with the major differences arising for larger frequencies. Similarly it can be shown by application of the Tauberian properties of Fourier transforms (i.e. determining properties of a function from behavior of its transform) that the values of the kernel function for small time are given by information obtained from its Fourier transform at large frequencies (see [18], [19]), as exhibited by

$$\lim_{t \to 0+} K_\theta(t) = \lim_{\omega_e \to \infty} i\omega_e \overline{K}(\omega_e) \tag{50}$$

Since the relations of Equation (43) essentially define the Fourier transform of the kernel function to represent the pitch transfer function relative to wave amplitude, the conclusions of this analysis are that the truncation of the kernel function is effectively equivalent to altering contributions of the higher frequency portions of the transfer function. Since the aircraft carrier has very little response at higher frequencies, this neglect will have little practical effect, and this is verified by the operations of physical realizability and truncation applied to the kernel functions obtained by transforming the frequency response (transfer) functions. Another interpretation of the effect of kernel function truncation in this manner will be discussed in a later section when considering the application of Wiener prediction methods, which is a statistical prediction method.

One of the significant factors concerning the ship transfer function characteristics in heave and pitch for an aircraft carrier of the Forrestal class was the fact that there appears to be only small differences due to the effect of heading between the ship and the waves, up to angles of 45° off the bow, when plotting the amplitude and phase characteristics as a function of the frequency of encounter ω_e [2]. Particular examples of this relation are shown in Figures 13 to 16, which exhibit the amplitudes of the heave and pitch transfer functions for forward speeds of 20 and 30 knots, for headings of 0° and 45°. Further information on these characteristics concerning the relative insensitivity to heading are given in [2]. As a result of this characteristic with regard to heading it is expected that the kernel functions defined by Equations (34) and (35) for heading angles (β) up to 45° will have only a small dependence upon the heading angle. An illustration of this effect is shown in Figure 17, which represents the kernel functions for two headings that differ by 45°. The basic difference is a small phase shift, which can be expected in accordance with the basic definition of the function $F(\omega_e)$ defined in Equation (4), which enters into the definition of the transfer function in accordance with Equations (30) and (31). Thus it appears that this insensitivity to heading will allow adequate prediction of ship motion for unidirectional oblique waves, when measuring the wave input at a fixed point ahead of the bow. Since complex short-crested seas are formed by a sum of various unidirectional irregular waves coming from various oblique headings, it can be expected that the prediction technique will function effectively in realistic seas that exhibit short-crestedness. Since the aircraft carrier will always aim to head into the wind during landing operations, most of the seas encountered will be bow seas as far as the predominant wave systems are concerned, and this technique will thereby be appropriate to realistic operational conditions.

All of the statements above are based upon the observed insensitivity of theoretical transfer functions to heading, but this must be verified by experiment before the technique can be expected to be useful in the manner described above. In order to obtain some information concerning these points, limited tests were carried out at the Davidson Laboratory of Stevens Institute of Technology [20] in oblique waves in order to obtain information concerning this point, as well as providing data that could be used for checking prediction capability using the derived kernel functions appropriate to oblique headings. During the course of the tests it was determined that the ship responses in regular head sea waves differed from those obtained during the MIT tests [14], which were previously shown to have good agreement with the theoretical transfer functions (see Figures 1 to 4). These differences may be due to different test techniques such as model support; method of ballasting; determination of model radius of gyration; method of wave measurement; etc., which are often responsible for differences in test results for the same model tested in different towing tank establishments. An indication of the differences between the theoretical and experimental amplitude responses in pitch for oblique headings (as well as head seas) is given in Figure 18, where the data was obtained at headings of 0°, 30°, and 45°. The most significant aspect of this figure, aside from the differences in magnitude between theory and experiment, is the relative insensitivity to heading of the pitch amplitude responses when plotted as a function of frequency of encounter. This limited data tends to support that particular contention of [2], and is thus useful for further development of the prediction technique described herein.

The data in oblique irregular seas was taken over a range of headings up to 45°, and for speeds up to 30 knots, in irregular sea states that were designed to have the same scaled significant height as Sea State 6. No analysis for motion reproduction or prediction was carried out for this data in the present preliminary study, due to the differences in transfer function characteristics obtained between the theory of [2] and the present experiments (as well as differences between experiments). As discussed previously, it is essential that the kernel functions be derived from valid ship transfer functions in order to expect good agreement in the time domain. It is therefore necessary to obtain further data on ship response characteristics at oblique headings, and to combine that information with available theory in order to obtain the representative kernel functions for purposes of prediction in various sea conditions. A source of extensive data will be available upon completion of the analysis of extensive model tests of an aircraft carrier at the David Taylor Model Basin [21], which will be used in a future program to develop the required prediction kernels.

The input data for the prediction technique developed herein is the wave height at a point ahead of the ship bow, and as such it will (ideally) be distorted by waves that are generated by the ship's dynamic motions (pitch, heave, etc.) as well as reflections of the oncoming waves by the ship itself. The wave elements that can distort the input signal are those waves that are propagated forward of the ship, and according to the theory of oscillating translating sources [22] such waves will exist when the dimensionless parameter $\frac{\omega_e V}{g} < 1/4$. In the case of regular waves the conditions for the occurrence of such waves can be determined precisely, but for an irregular pattern there may be some wave energy in a frequency band that could result in waves propagated forward. However for the speed range of aircraft carriers, which have been considered in this study, there appears to be very little significant wave distortion due to the limited oncoming wave energy in the necessary frequency band (e.g. for V = 20 knots, ω_e values < .238 are necessary for the occurrence of forward propagated waves). This lack of significant wave interference is supported by the experimental data obtained in this program, where wave height data was simultaneously measured by a transducer located far to the side (at the same x-coordinate location) and limited comparisons made with the bow wave transducer data. Similarly the lack of any significant interference may also be inferred from the results shown by the predictions of Figures 10-12. For lower forward speeds it is possible that some wave distortion will occur, and it will be necessary to determine the effects of such distortion on the prediction process. This will be a subject for study in future work on this program.

WIENER PREDICTION THEORY

The technique used for prediction of a stationary time series (of which ocean waves and ship motions may be considered representative) by statistical methods is most often that developed by Wiener [8]. This technique determines an optimum linear predictor on the basis of minimizing the mean-square prediction error, using integral equations as the analytical tool. The final form of the solution to the prediction problem developed by this method is a linear mathematical operation on the past history of the signal, which will yield a short time prediction in the future. The signal that must be predicted is a random signal, which is only characterized by certain statistical parameters. The most important statistical characteristic required for this method is a knowledge of the power spectrum of the desired random function.

Assuming a complete knowledge of the power spectrum of the signal, this function must be approximated as a rational function of frequency. The most crucial operation is the spectral factorization of the power spectrum into a product of two conjugate functions, and the selection of the portion of an integrand function which will have no poles (in the complex variable sense) in a certain region of the complex frequency plane. The transfer function (frequency response) of the optimum predictor is readily determined from this last result. The predictor transfer function (for prediction time T), denoted as $J_T(\omega_e)$, is a complex function of frequency which must be compared to the ideal prediction transfer function $e^{i\omega_e T}$ (obtained from the translation theorem of Fourier transforms [17]) so that the error

$$E_T = \frac{1}{\pi} \int_{-\infty}^{\infty} \Phi(\omega_e) | J_T(\omega_e) - e^{i\omega_e T} |^2 d\omega_e \quad , \tag{51}$$

where $\Phi(\omega_e)$ is the power spectrum of the random variable to be predicted, will be a minimum. It can be shown that this method results in an inherent error that increases as the prediction time is increased, and which only depends on the power spectrum of the random variable input to the predictor. Other significant aspects of this prediction technique are the physical realizability property of $J_T(\omega_e)$, which is ensured by the procedure, and the restriction that there is no extraneous noise in the recorded signal (the presence of noise in the input would lead to a more complicated transfer function that would require the combined operations of filtering and prediction).

The required operations for a representative random process are too extensive to be outlined in this report, and details can be obtained from the pertinent literature, e.g. [9], [10]. Particular applications of this technique to aircraft carrier motions, wave records, and related low frequency phenomena are given in [23] and [24]. The results of these latter studies indicate a fair degree of pitch prediction for the order of 5-6 seconds, with the error increasing significantly if the prediction time is increased much beyond this.

The main difficulty in implementing the Wiener prediction techniques described in the foregoing is the necessity for complete and accurate knowledge of the power spectrum of the signal to be predicted. The determination of power spectra of low frequency random processes is a relatively complex operation, and requires careful processing of observed data for a time interval of approximately 20 minutes in order to obtain an accurate representation [25]. The power spectrum must be represented as a rational function of frequency, on which the processes of spectral factorization and subsequent mathematical operations must be performed in order to arrive at the form of the transfer function representation of the predictor. The required processing equipment for these operations is relatively complicated and expensive, and the final implementation of the predictor circuit in the form of an electrical network or analog computer system will also be difficult to achieve, if these operations are to be carried out on an aircraft carrier for on-line prediction purposes. This last statement is true even if the power spectrum of pitch motion (the main variable of interest in this study) does not change significantly for a period of a few hours. However, there are significant changes in the power spectrum of pitch motion due to the effects of varying oceanographic conditions; changes in forward speed (which also effect the frequency of encounter); heading variations; varying wind conditions; etc., under realistic operational conditions, and the implementation of a predictor is further complicated by these effects. In view of the above, it appears that the use of Wiener prediction techniques has only limited application to aircraft carrier motion prediction, unless certain more complicated methods, such as development of an adaptive predictor, are to be utilized. In view of the complexity of such a development, this approach will not be pursued in the remaining portions of this report.

Since the prediction technique using the truncated kernel has been shown to result in good prediction capability when operating upon the measured waves as an input, it appears possible to extend the prediction time of ship motion by obtaining a predicted wave input and operating upon this enlarged input data with the known kernel function predictor. Since the kernel function effectively acts in such a way to filter certain portions of the input data (i.e. the results appear to be somewhat independent of higher frequency contributions), it is possible that prediction of the wave input can be made with sufficient accuracy. Then the composite operation of use of the kernel function operating on a wave input made up of the previous wave history, together with a predicted value for a few seconds (≈3 seconds ahead), can result in an enlarged prediction time of the order of 9 seconds. The basis for expecting a simple treatment of prediction of the wave input is the fact that the most significant parameter required for adequate prediction by the Wiener technique is a knowledge of the central frequency, i.e. the frequency for the largest value of spectral energy, according to the results of [24]. It appears easier to determine this particular characteristic, and the approach is further enhanced by the fact that required accuracy of matching other parameters such as the bandwidth, the high frequency cutoff rate, etc., will not be so important due to the ultimate filtering action of the kernel function representation. On this basis a representation of the power spectral form of a Sea State 6 head sea wave system, appropriate to a 20 knot forward speed, was established, and a mathematical fit to the form of the spectrum was obtained with high accuracy by means of the Chebyshev filter approximation. This complicated form of power spectrum representation was factorized, and the Wiener prediction procedures applied for determination of predicted output. The resulting expression for the optimum predictor was determined, which behaved as a combination of a proportional plus derivative operation at high frequency, and a triple integral operation at very low frequencies, i.e.

$$\lim_{\omega_e \to \infty} J_T(\omega_e) \approx c_1 + c_2 i\omega_e \quad , \quad \lim_{\omega_e \to 0} J_T(\omega_e) \approx \frac{c_3}{\omega_e{}^3} \; . \tag{52}$$

The behavior at high frequencies can be altered in order to insure a frequency response that attenuates at large frequencies, so that the resultant spectral energy is finite. An alternate approach was used whereby the form of the predictor was not chosen to be the optimum predictor, but one that would have a minimum mean square error for a prescribed form that would be simple to deal with analytically and in regard to actual electrical network construction. That particular form is given by

$$J_T(\omega_e) = \frac{a + ib\omega_e}{1 + i2\zeta_n \frac{\omega_e}{\omega_n} - \frac{\omega_e{}^2}{\omega_n{}^2}} = J_T(\omega_e;a,b) \tag{53}$$

where the value of the damping coefficient ζ_n and the natural frequency ω_n are chosen initially to supply a flat response in the frequency range of significance for the variable. The unknown coefficients a and b are determined by minimizing the error

$$E_T(a,b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\omega_e)|e^{i\omega_e T} - J_T(\omega_e;a,b)|^2 d\omega_e \tag{54}$$

which is easily obtained by applying partial differential procedures (with respect to a and with respect to b) in Equation (54). This procedure was also formulated and applied to the same Sea State 6 wave input. The computations necessary for evaluation of the resulting predictor are also quite complicated, and the final expression will not be given here. It is sufficient to remark that the complexity of the operations in both of the above representations was mainly due to the complicated Chebyshev representation of the wave power spectrum, and it is expected that simpler representations in the future will yield more useful results in an easier manner. The forms that will be obtained in that case must be checked in detail to determine the sensitivity to the central frequency value, effects of high frequency attenuation, and other factors that will influence the final form required with the minimum number of parameters for effective prediction by this approach. It appears however that this is a more fruitful utilization of prediction techniques for application to the overall problem of aircraft carrier motion prediction, whereby the properties of the kernel function overcome some of the inherent errors in the wave motion prediction.

In the course of investigating the techniques of Wiener prediction, certain aspects of the procedures used in establishing a predictor by the formal methods described in [9] and [10] appeared to be similar to the action of truncating a kernel function and shifting to a new origin, as indicated in Equation (41). The formal establishment of the predictor by the Wiener technique, as modified by other investigators (see description in [9] and [10]), requires this alteration in an effective kernel function that is derived from the power spectrum of the random signal that is to be predicted. In particular, it can be shown that if the wave input to a ship is white noise of unit spectral energy in magnitude, then the formulation of the prediction operation according to the Wiener theory gives exactly the same result as the use of the truncated kernel technique, when the kernel function acts on waves with a white noise spectrum Since the wave input will have an arbitrary spectrum that is certainly not white noise in almost every possible situation of interest, this similarity noticed herein may only be of limited significance. However, any possible implications that result from this similarity, and also the actual meaning of these operations will be deferred for future study and detailed analysis, in view of the limitations imposed by the preliminary nature of this study.

POSSIBLE EXTENSIONS OF PREDICTION THEORY

In view of the fair degree of success shown in the previous sections in predicting carrier pitch motions up to 6 seconds ahead, as well as the possibilities inherent in other prediction techniques applied to the waves, it appears that this technique should be studied further in order to ultimately arrive at a practicable prediction technique for operational evaluation in a full scale installation. The ability of the kernel function prediction method, which is deterministic in nature, to be applied to oblique headings should be determined in more detail with the use of experimental data for purposes of verification as well as for guidance in establishing the proper mathematical functions to be used. Upon completion and satisfactory determination of the appropriate kernel functions for oblique headings, which should not differ significantly from those obtained in head seas (according to the theory developed in [2] as well as the present study) the problem remaining is that of a technique for real-time computer processing of the truncated kernel predictor method described in an earlier section of this report. In addition, it is necessary to have some means of supplying the wave motion time history at the bow as the basic input to the predictor, and it is fortunate that a device previously developed for the Navy Department can serve this purpose. The particular device is a wave height sensor which uses ultrasonic waves to determine the surface wave time history, with appropriate compensation networks to correct for the motion of the transducer itself, since it is intedned to be located at the bow of a ship which is translating and oscillating [26]. This device has the required accuracy for determining a continuous wave time history as input data, which can then be used with known kernel functions (stored in a digital computer memory) in order to carry out the required mathematical operations. A limited storage of kernel function data is necessary (only for approximately 36 seconds of time) and the necessary computations can be easily carried out by a small high speed general purpose digital computer or by a special purpose computer designed for this particular application. The availability of the wave measuring device, as well as recent developments in high speed computation, are important aspects of the proposed prediction approach based on the kernel function technique.

With regard to prediction of the wave motion input, which has been described as a possible application of Wiener prediction techniques, an alternate method may be used for prediction purposes that appears to be more fruitful both conceptually and for direct application using the digital technique described above. The method of prediction that may be used for the wave input is an application of the recently developed techniques of recursive filtering and prediction applied to a sequentially sampled digital input [27]. This new technique, using the so-called Kalman filter, is applicable to both continuous data and sampled data, and it appears that the proposed digital format with sampled inputs will be appropriate for this problem as long as suitable digital-to-analog conversion equipment is available. The determination of the fundamental transition matrix for the Kalman filter approach will be studied in a similar manner as the concepts for the Wiener predictor described in [24], which is also discussed in the previous section of this report, in order to determine the least number of parameters for successful prediction of the wave time history.

Following the development of this particular approach, it is expected that a simple composite operation based on wave input data alone being fed into a computer that contains a pre-programmed kernel function, as well as other simple operations that may require a selective input (such as forward speed, or observed central frequency, etc.) will be the final technique appropriate to the overall procedure. In view of the success shown by the results of the preliminary study described in this report, this program appears to offer promise of useful prediction capability when implemented in a full scale installation under operational conditions.

CONCLUSIONS AND RECOMMENDATIONS

The major result of this study is the development of a means of calculating ship motion time histories, including predicted values for a short time ahead, by means of a convolution integral representation. A kernel function, derived from ship response functions, is modified by certain truncation operations and is then applied to operate on input data in the form of the present and past history of wave motion measured at the bow of the ship. A detailed derivation of this significant relation is presented which accounts for the spatial influence of wave motion, the effect of oblique heading, and which interprets the final form of the kernel function in terms of the frequency characteristics of ship responses. The derivation and resulting interpretation of the results is more general than in previous studies in the time domain, and an effort is made to relate the representations in the two domains of time and frequency.

An outline of classical Wiener prediction techniques applied to this program of aircraft carrier motion prediction is presented, and the limitations as well as the practical difficulties of implementation of such an approach are presented. The significant features of the kernel function technique, in contrast with a statistical technique such as the Wiener method, are shown to be an independence of the statistical characteristics of the waves and the ship motion itself; small dependence upon heading angle in the practical operational regime; ease of implementation using a high speed digital computer; and a limited dependence on forward speed which can be overcome by pre-programming speed-dependent effects on the kernel function in the computer memory.

The direct application of the kernel function deterministic prediction method yields a prediction time of about 6 seconds, and means of increasing this time are considered. An attractive approach appears to be the use of modern prediction techniques, such as the Kalman filter method [27], applied to the wave motion input so that the kernel function will tend to smooth some of the prediction errors inherent in that approach. In view of the proposed digital format of the basic prediction method using the kernel function method, this technique can be readily incorporated into the overall procedure. Thus the prediction time can possibly be extended up to about 9 seconds, with fair accuracy, by this proposed method. It is necessary to develop in detail the basic prediction formulas for wave motion data, and to check the accuracy of that approach with recorded data. Similarly more data on ship motion and its degree of agreement with predicted values should be analyzed, especially for oblique headings. All available experimental and theoretical information should be utilized to develop the final form of the applicable kernel functions for this purpose.

The predicted ship motion data, when obtained in full scale on board an aircraft carrier at sea, can be incorporated into the landing operation. In application to manually controlled landings it will serve as a valuable aid to the Landing Signal Officer (LSO) for earlier and more precise determination of wave-off criteria associated with landing on a moving deck. The information will similarly be useful in the SPN-10 Automatic Carrier Landing System by providing more advanced information on terminal conditions and thereby contributing to the reduction of landing accidents.

It is anticipated that successful performance of the prediction system described in this report will increase the safety of the landing operation and will also extend the range of sea conditions in which carrier operations will be possible. A system such as that proposed herein has a greater possibility of success, and appears easier to implement, than other suggested means for controlling the ship motion environment such as addition of anti-pitching fins [28], [29] which only have limited motion reduction capabilities; introduce additional disturbances; and/or require fitting the ship with large appendages, additional powering units, etc. Thus the practical implementation means of the proposed prediction system is another factor in its favor as a method for mitigating the influence of the sea environment on the landing operation. The success achieved so far, as well as the prospects indicated as a result of further development effort, offer important support for continued investigation of this technique.

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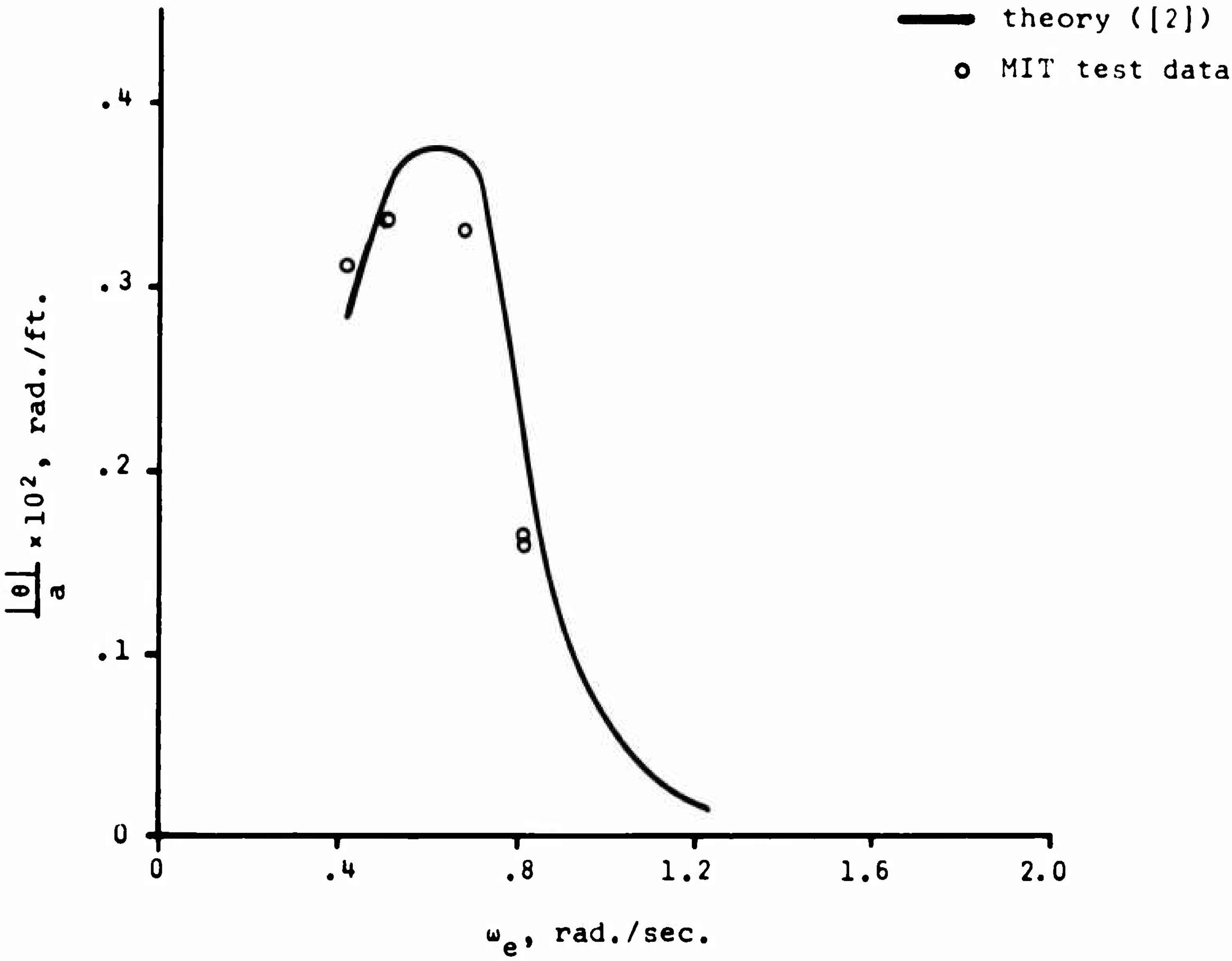

Fig. 1 Comparison of full scale theoretical and experimental pitch amplitude response in head seas, V = 20 knots

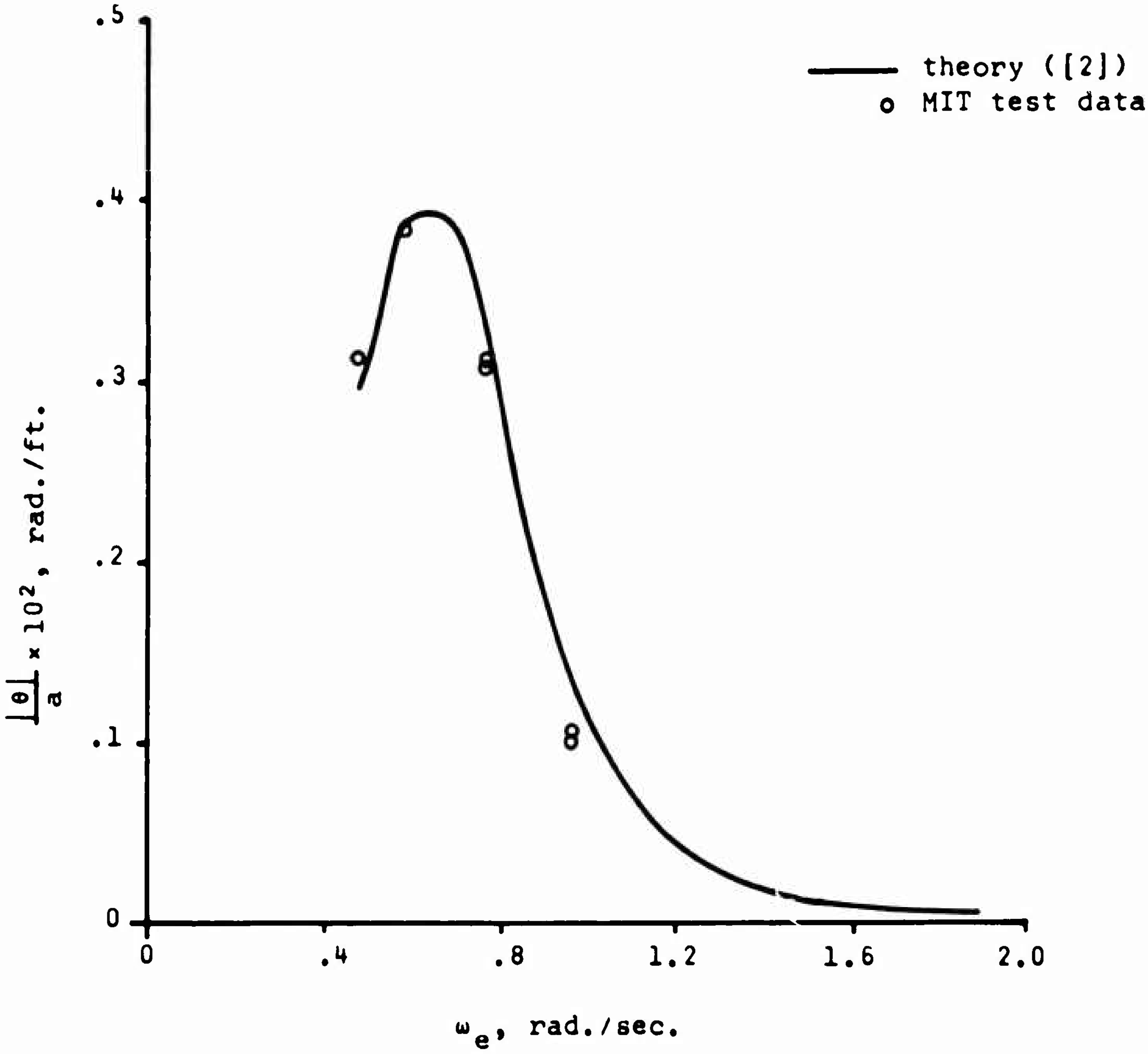

Fig. 2 Comparison of full scale theoretical and experimental pitch amplitude response in head seas, V = 30 knots

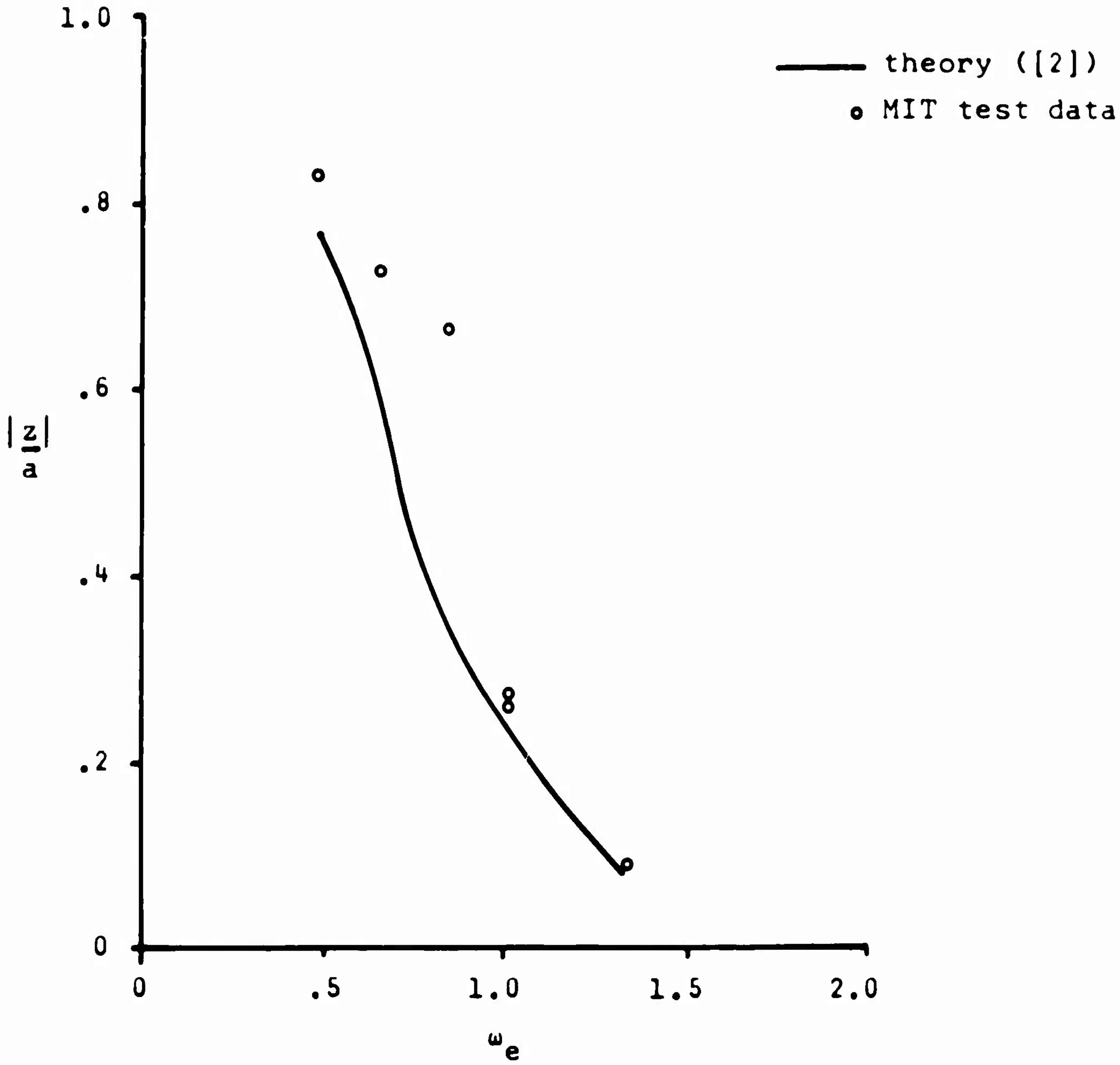

Fig. 3 Comparison of full scale theoretical and experimental heave amplitude response in head seas, V = 20 knots.

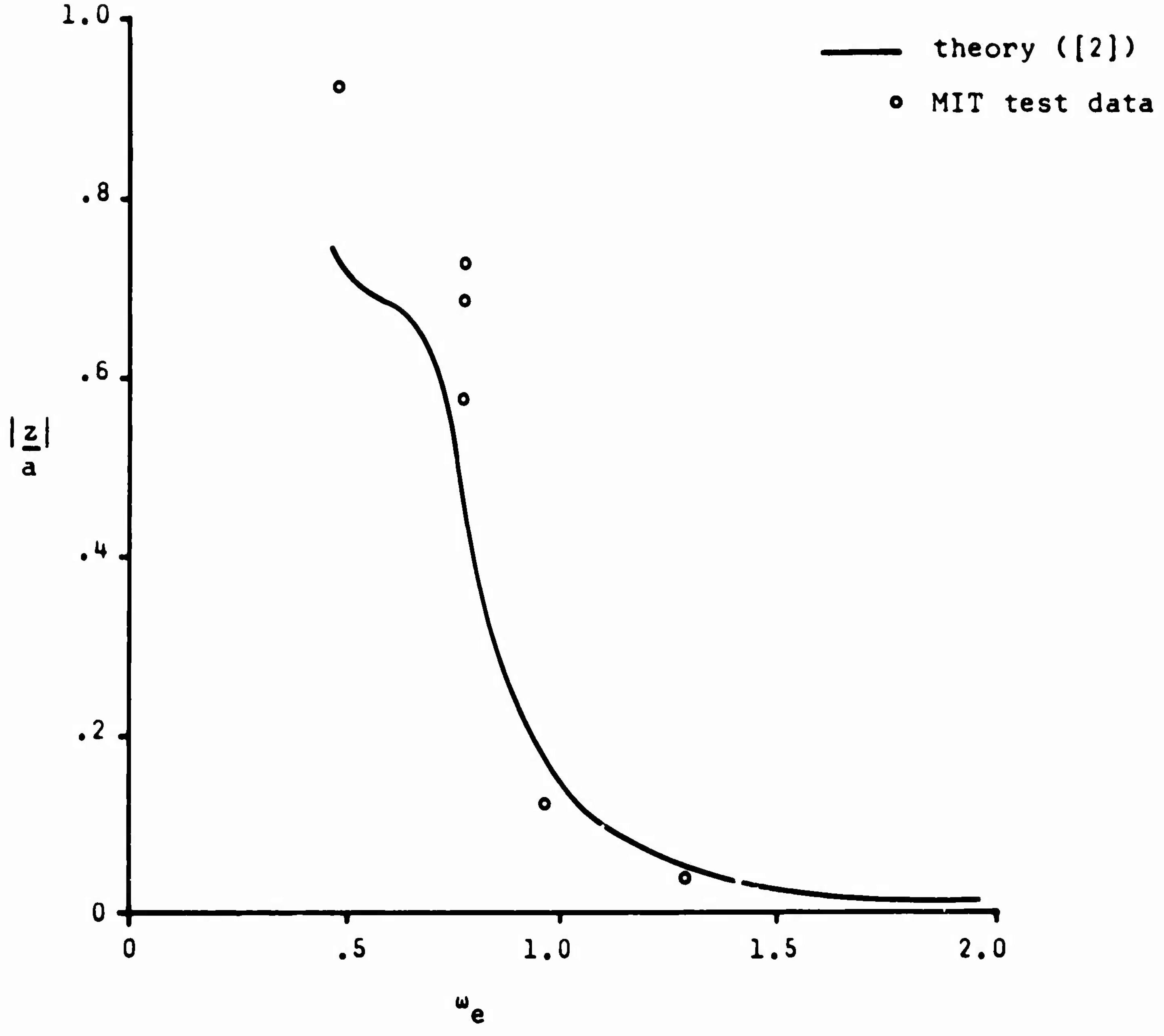

Fig. 4 Comparison of full scale theoretical and experimental heave amplitude response in head seas, V = 30 knots

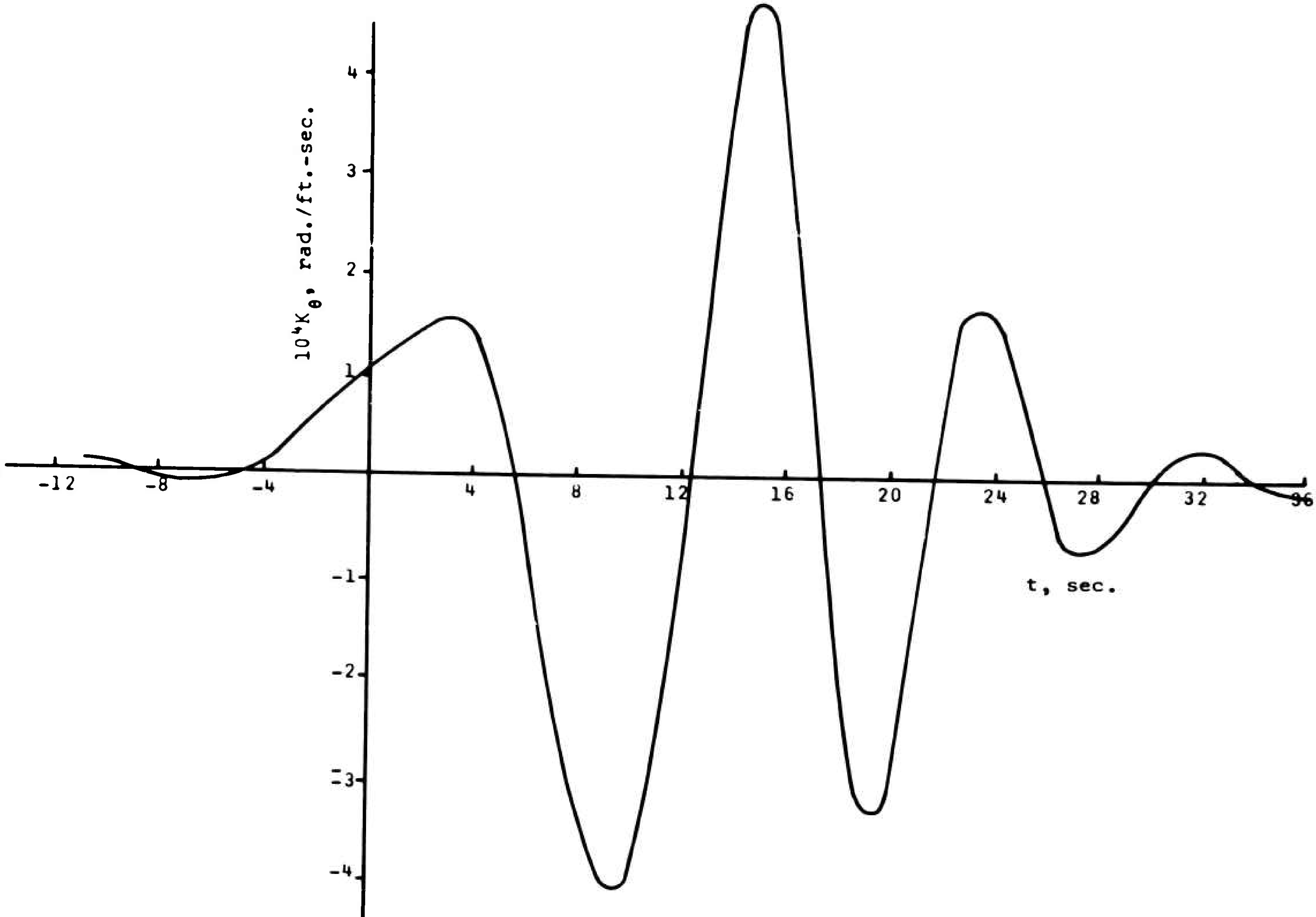

Fig. 5 Full scale pitch kernel function for head seas, V = 20 knots

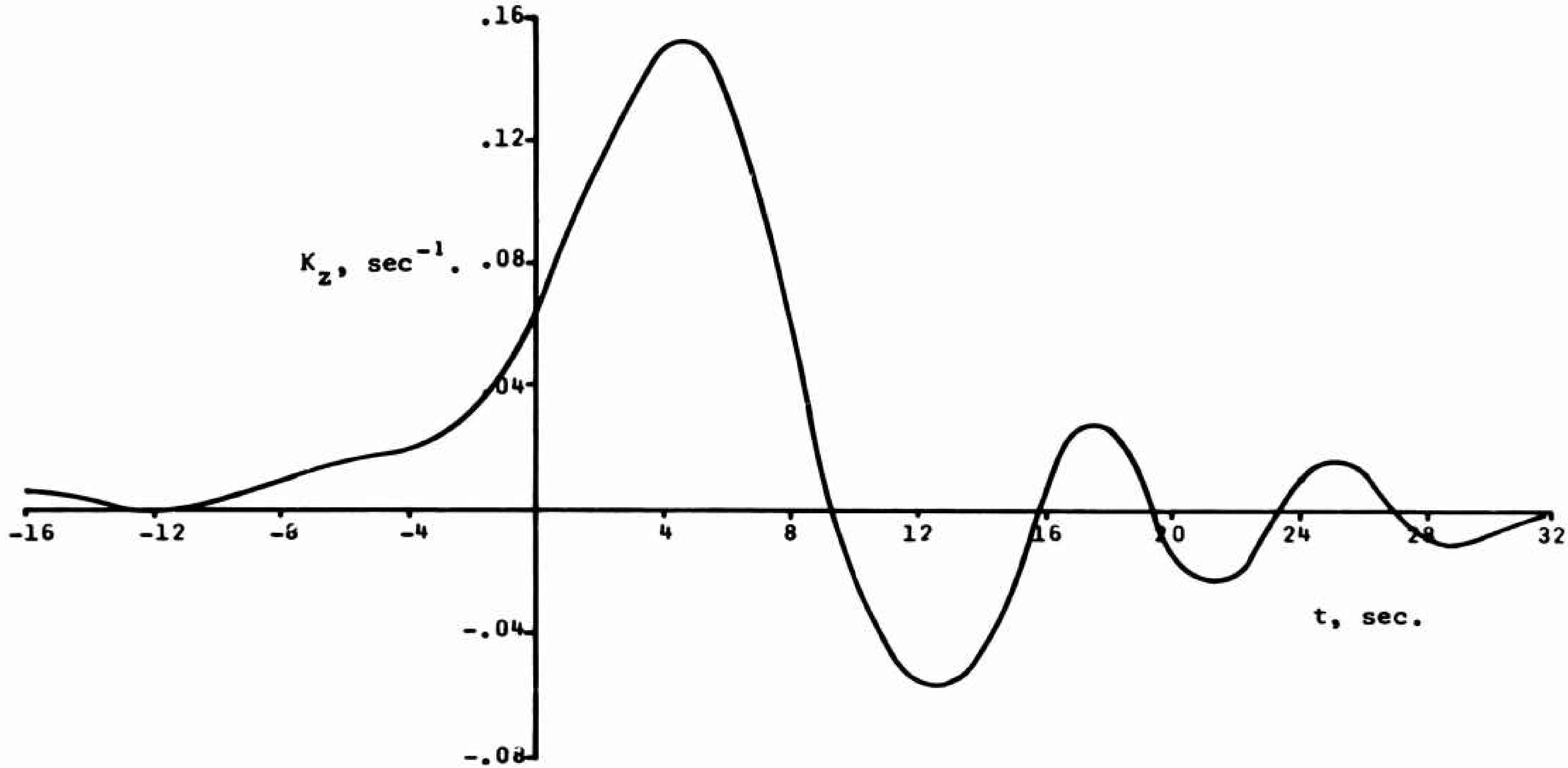

Fig. 7 Full scale heave kernel function for head seas, V = 20 knots

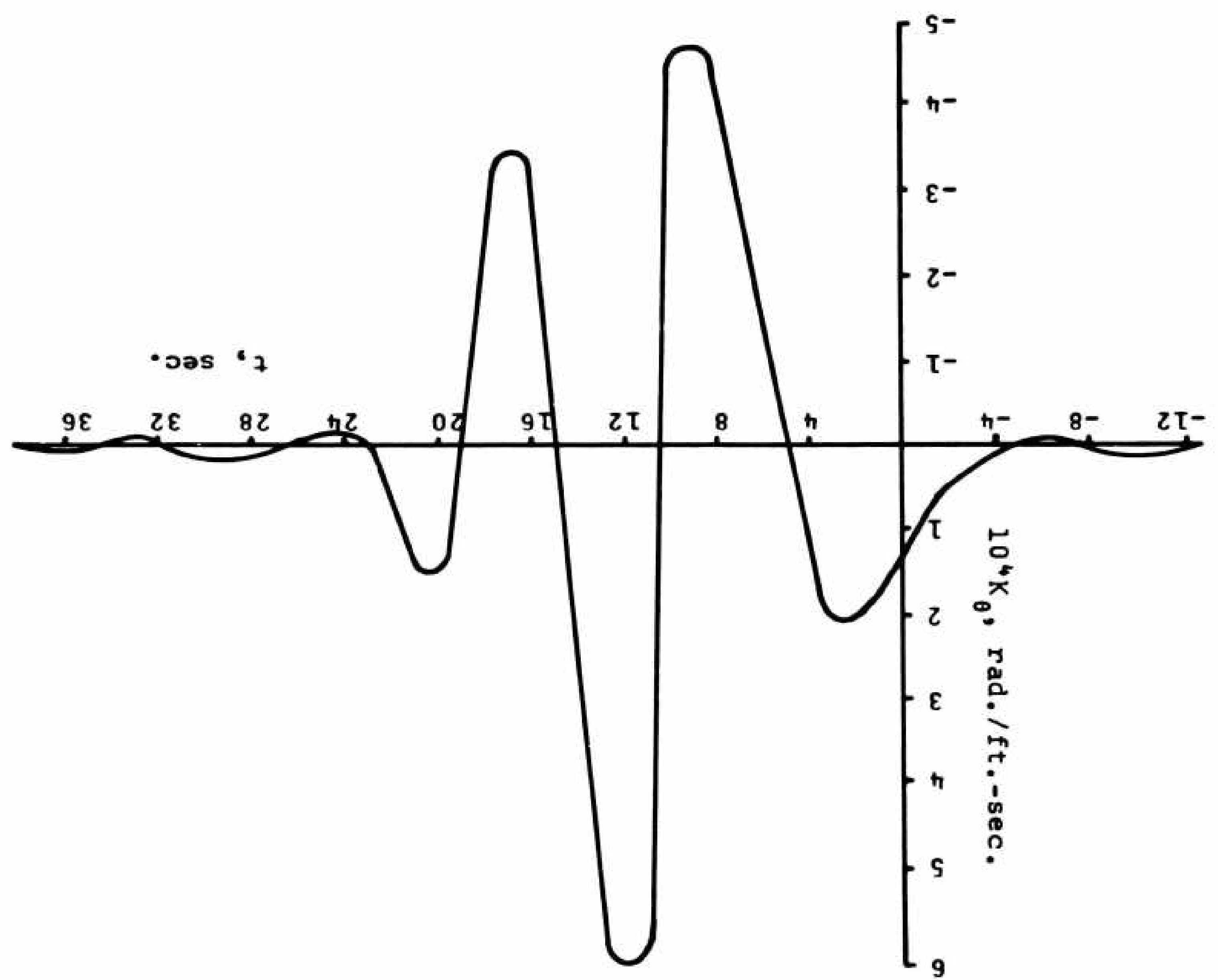

Fig. 6 Full scale pitch kernel function for head seas, V = 30 knots

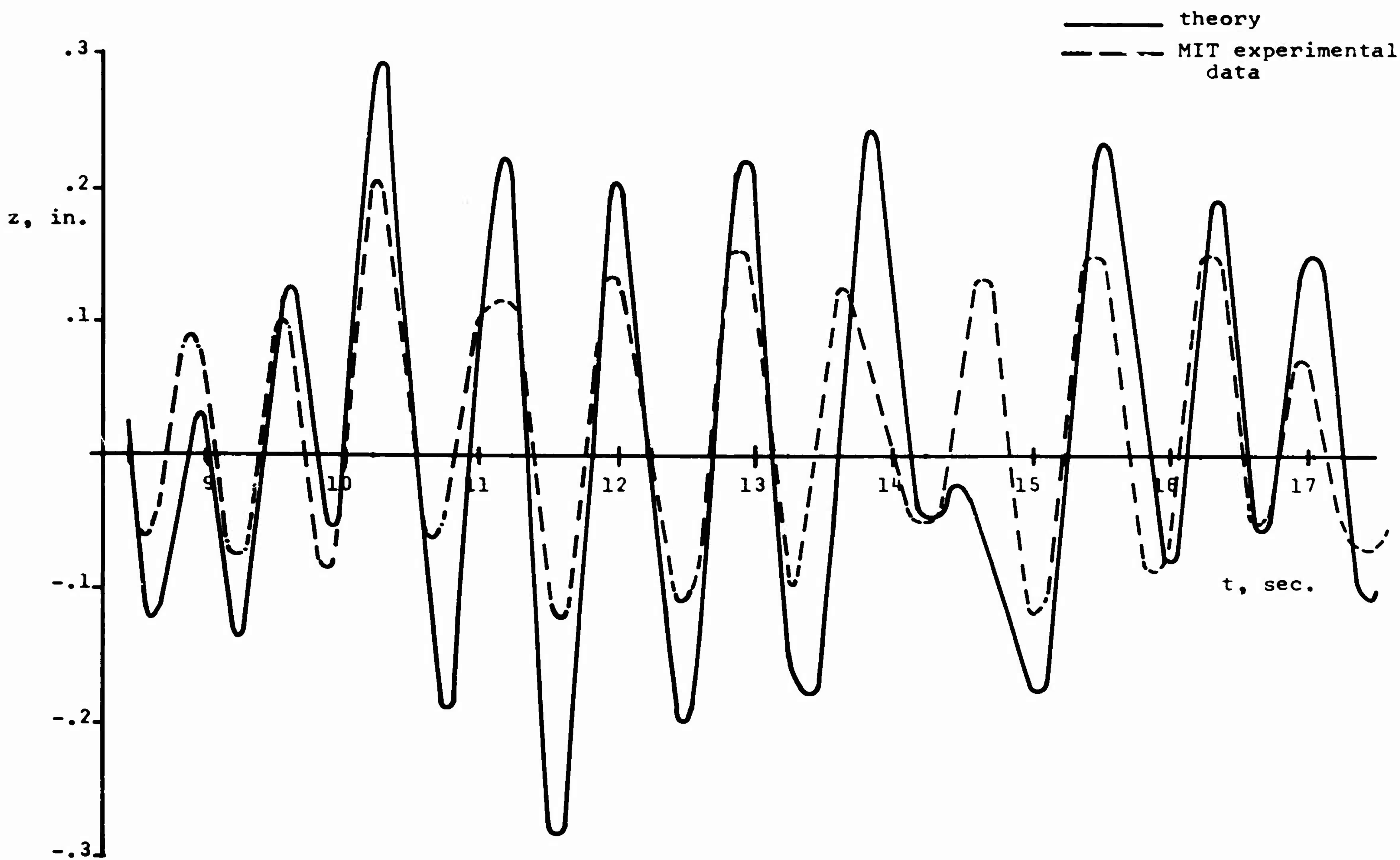

Fig. 9 Comparison of theoretical heave motion reproduction with experimental model data for head seas, V = 20 knots

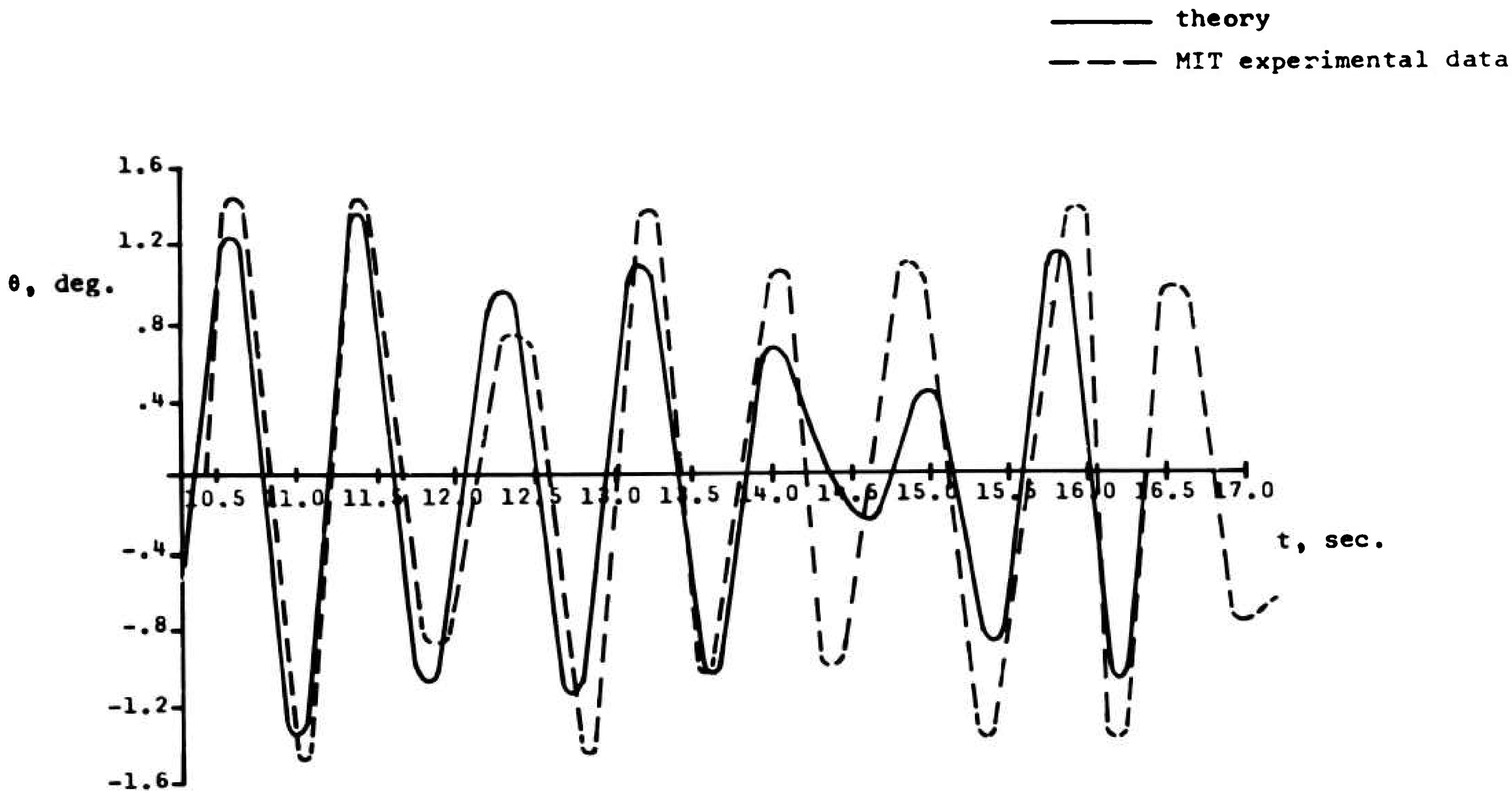

Fig. 8 Comparison of theoretical pitch motion reproduction with experimental model data for head seas, V = 20 knots

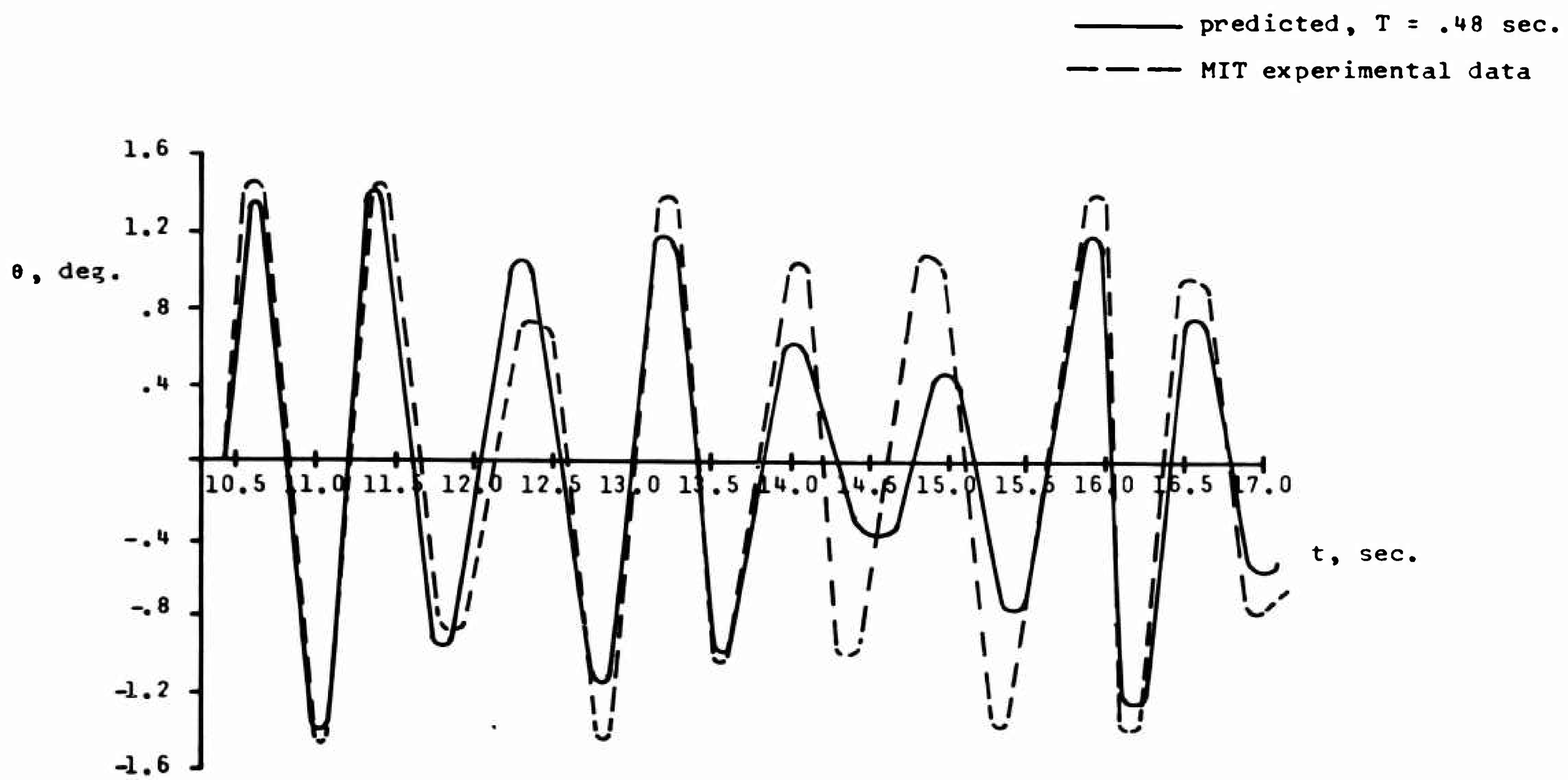

Fig. 11 Comparison of predicted pitch motion with experimental model data for head seas, V = 20 knots, prediction time T = .48 sec.

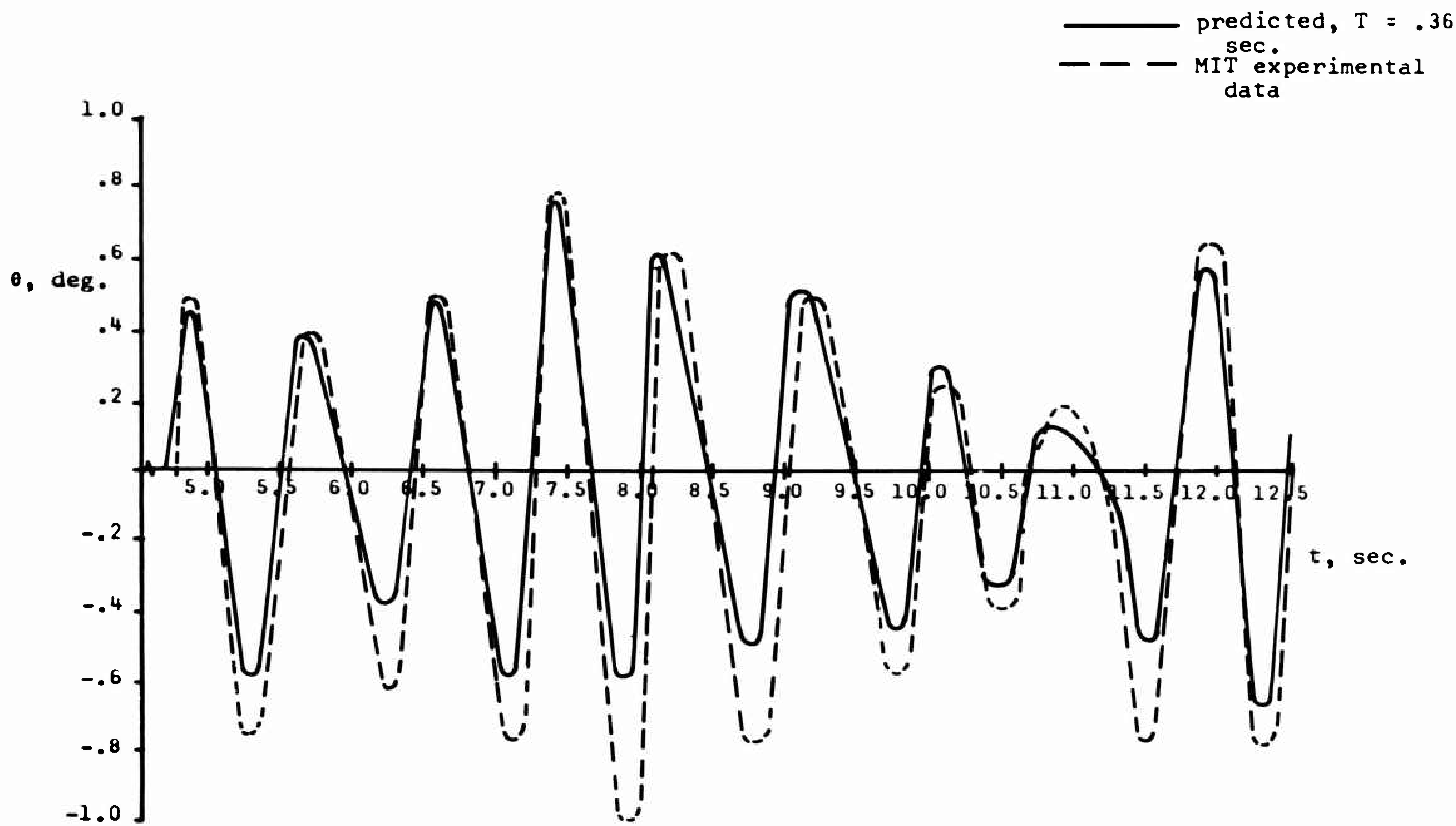

Fig. 10 Comparison of predicted pitch motion with experimental model data for head seas, V = 20 knots, prediction time T = .36 sec.

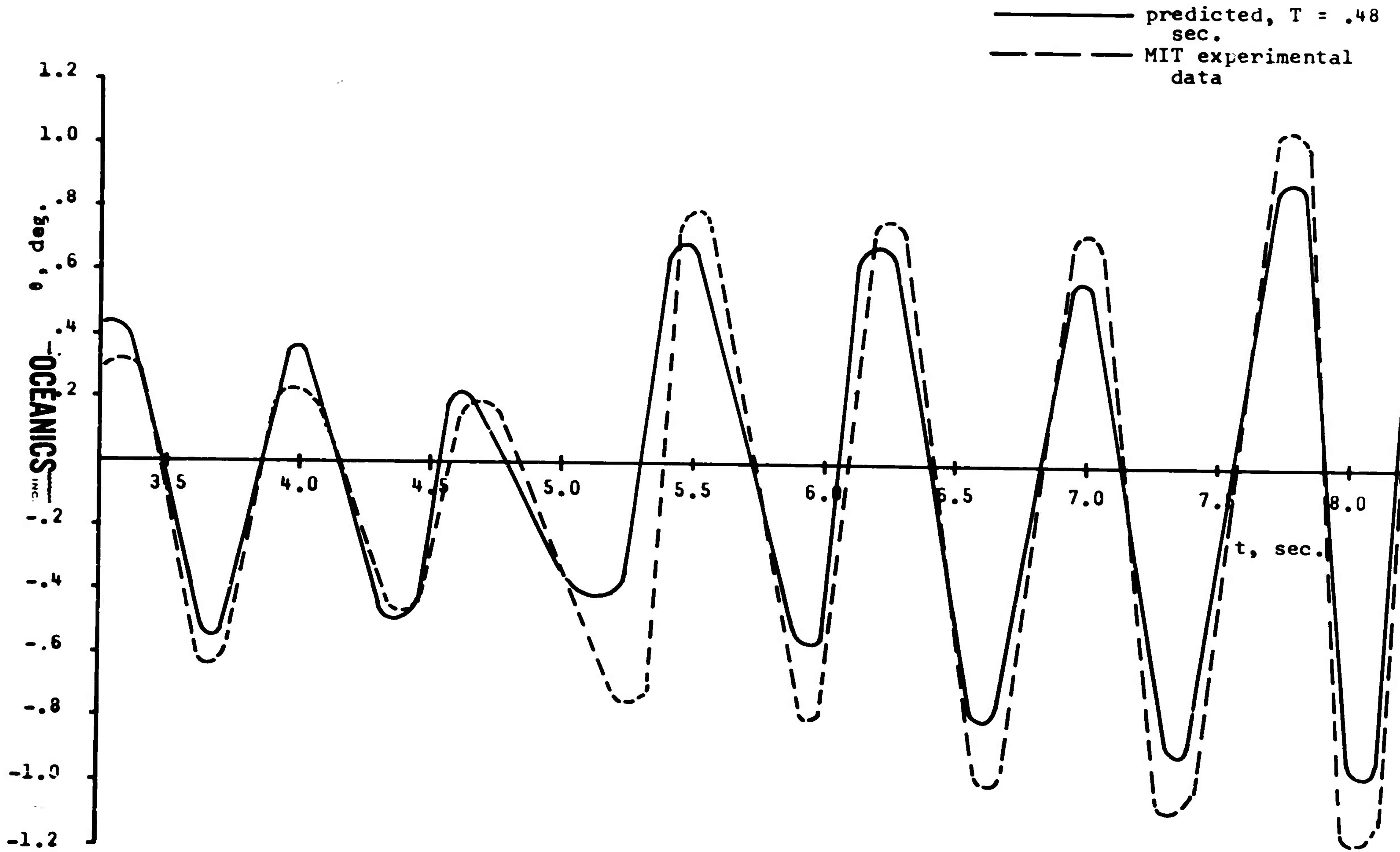

Fig. 12 Comparison of predicted pitch motion with experimental model data for head seas, V = 30 knots, prediction time T = .48 sec.

V = 20 knots

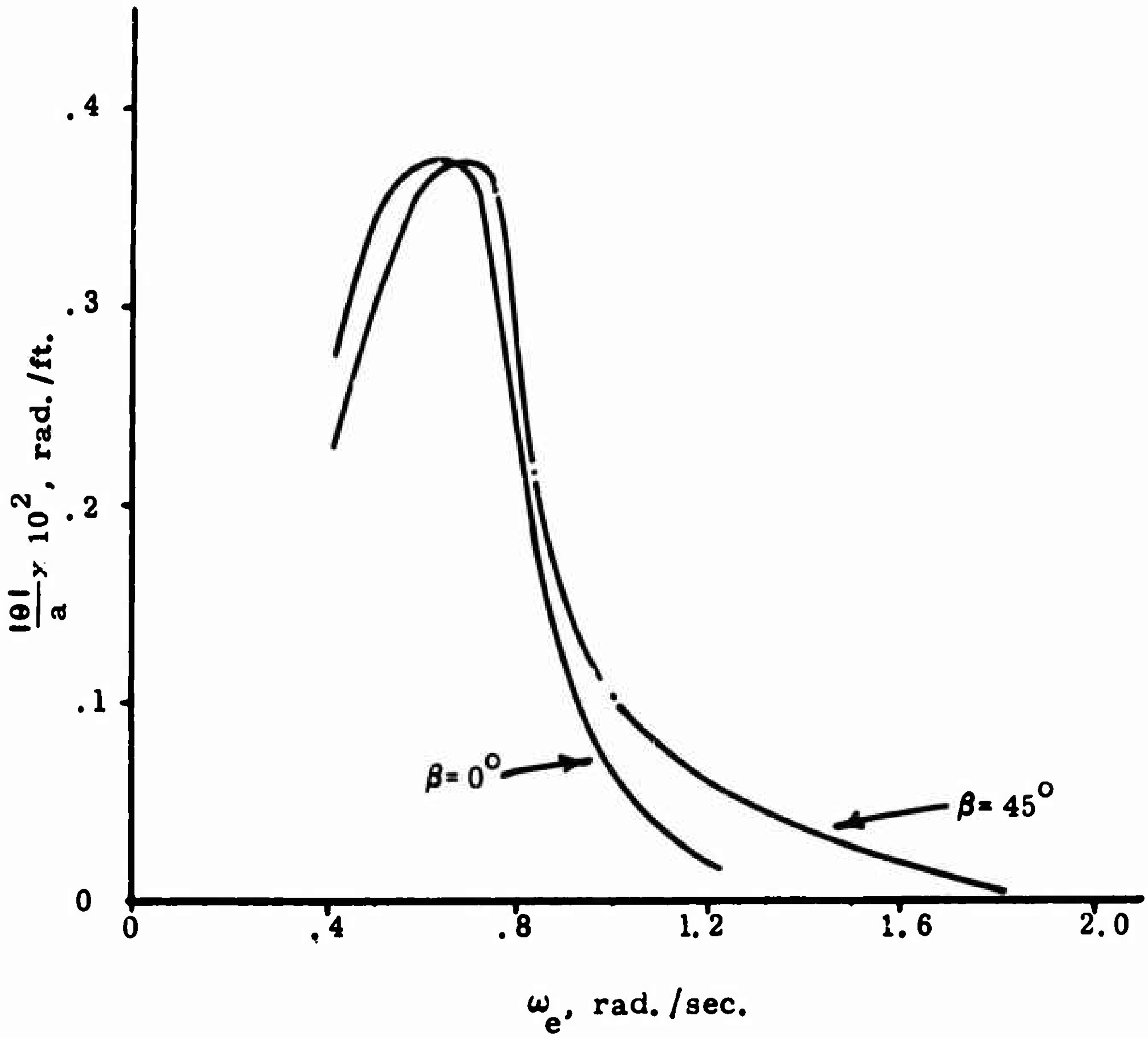

Fig. 13 Pitch amplitude variation with encounter frequency and heading, V = 20 knots

V = 20 knots

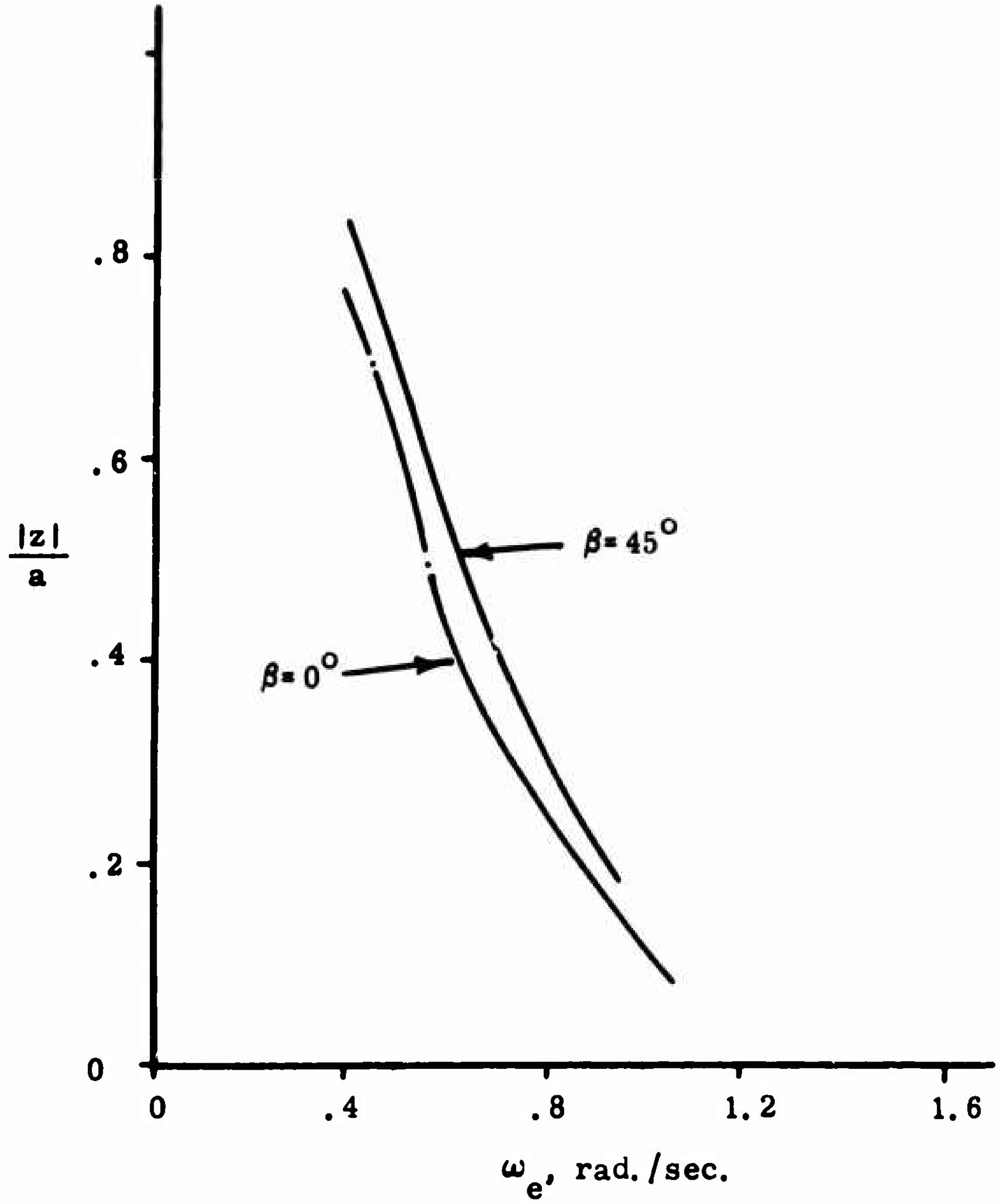

Fig. 14 Heave amplitude variation with encounter frequency and heading, V = 20 knots

V = 30 knots

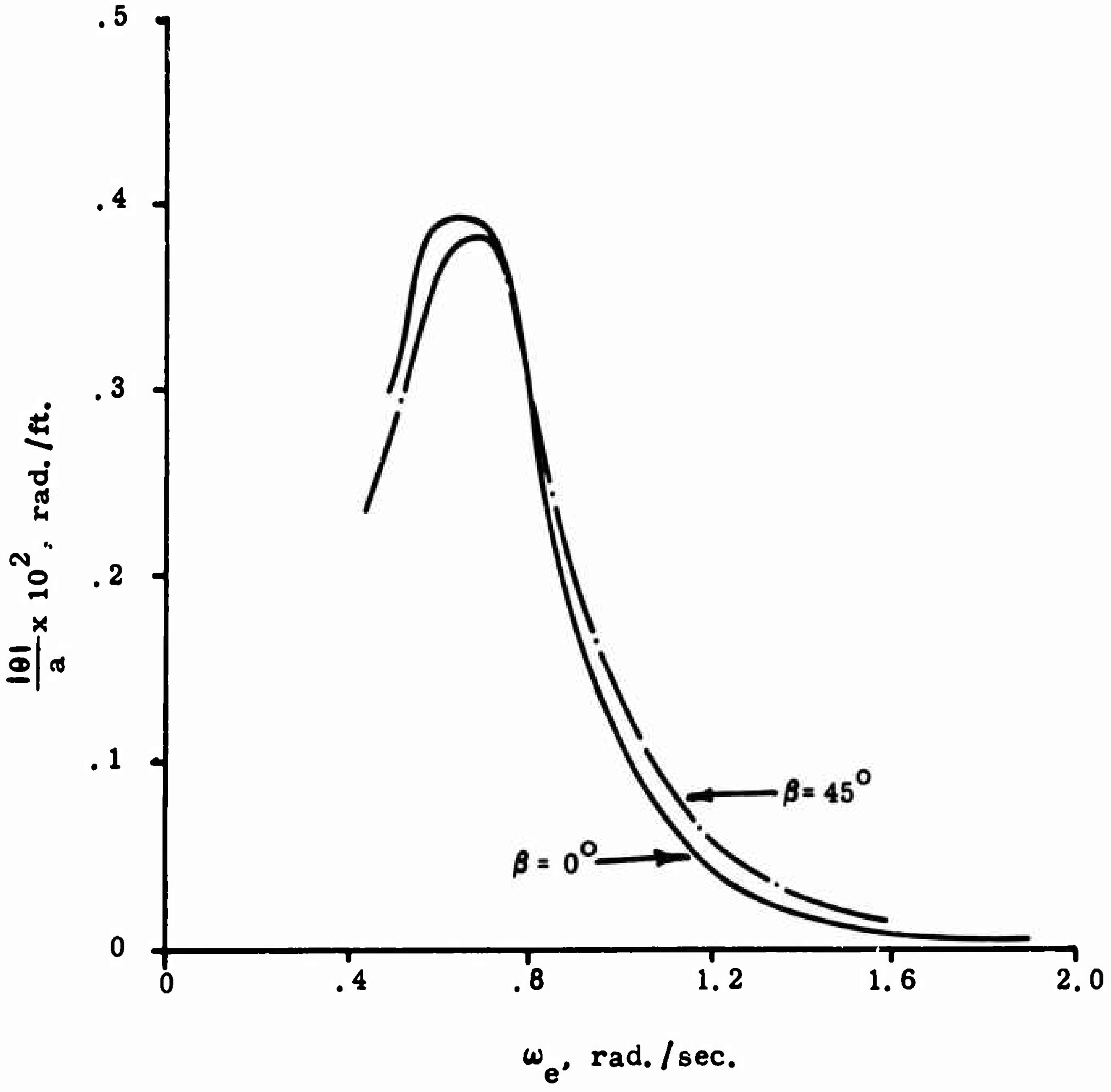

Fig. 15 Pitch amplitude variation with encounter frequency and heading, V = 30 knots

V = 30 knots

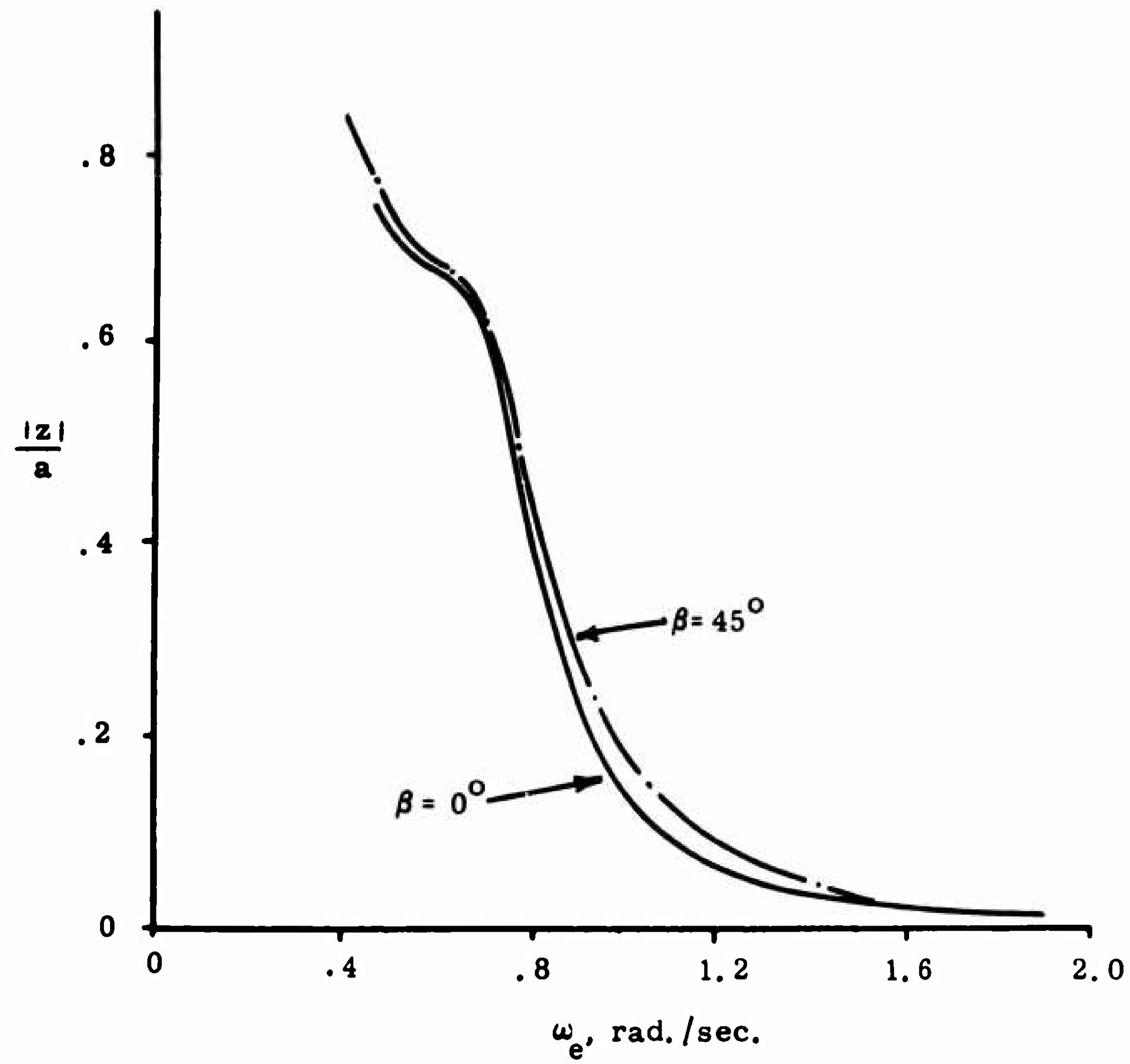

Fig. 16 Heave amplitude variation with encounter frequency and heading, V = 30 knots

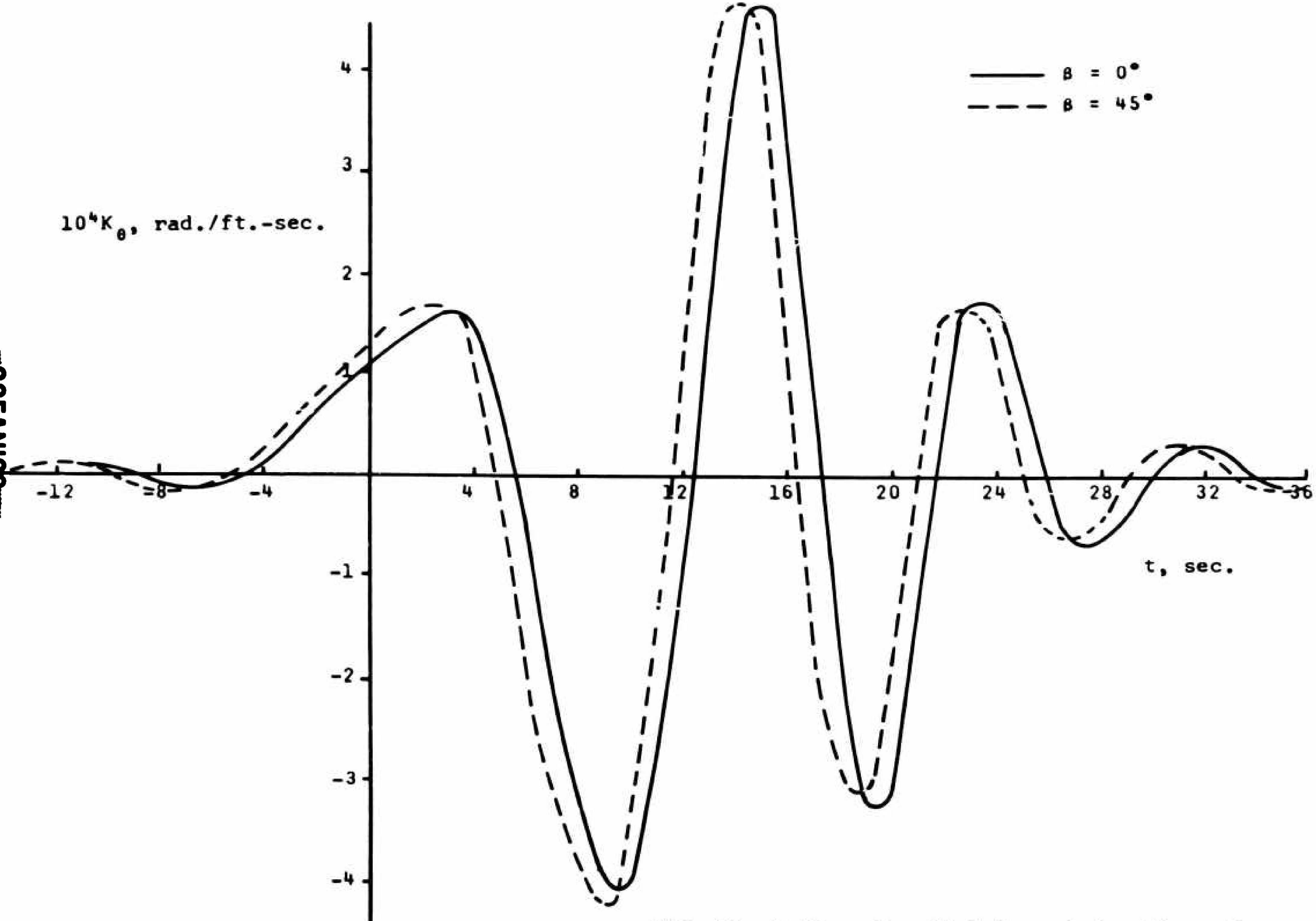

Fig. 17 Full scale pitch kernel functions for different headings, V = 20 knots

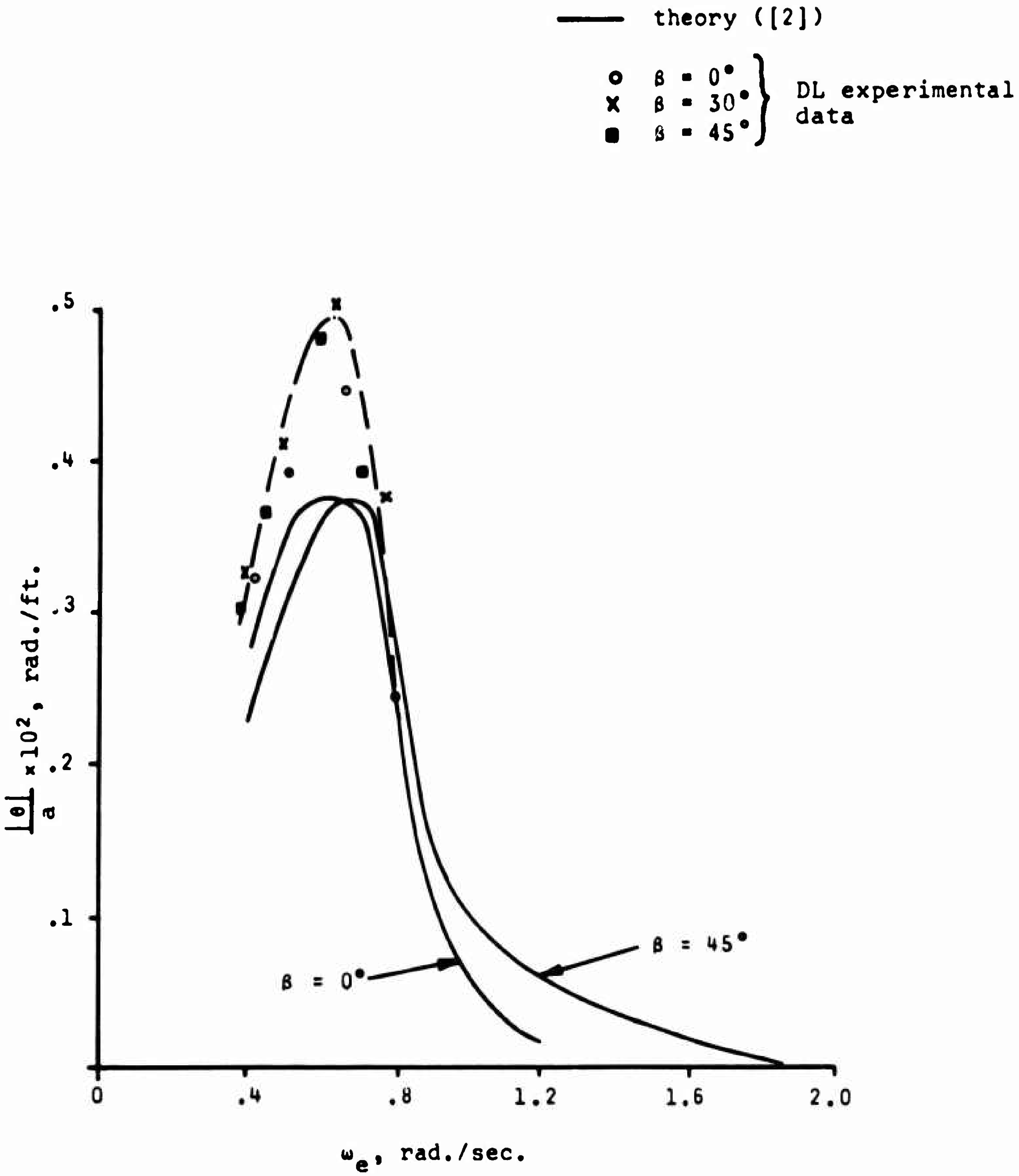

Fig. 18 Comparison of theoretical and experimental pitch amplitude responses for different headings, V = 20 knots

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13 ABSTRACT

Mathematical techniques for calculating ship motion time histories are developed for application to the aircraft carrier landing operation. Various methods for short-term prediction of motion time history are considered, based on both deterministic and statistical techniques. The most attractive approach for prediction purposes is the deterministic technique based on a convolution integral representation, with wave height measurements at the bow serving as the input data. A kernel-type weighting function, which operates on the input to provide the predicted motion history, is derived from ship response functions, and is shown to yield good pitch prediction up to 6 seconds ahead.

The limitations of classical statistical prediction techniques, as well as practical implementation difficulties, are exhibited. Certain aspects of recent prediction theory developments are considered for a proposed hybrid prediction technique (i.e. containing elements of both deterministic and statistical approaches) that will be compatible with the envisioned digital format of the predictor, and which will increase the prediction time. Recommendations for specific areas of further investigation are given for extending the applicability of the methods developed in this study.

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